A Novel Method to Measure the Zero-Frequency DQE of a Non-Linear Imaging System

Michael C. McDonald\textsuperscript{a}, Ho Kyung Kim\textsuperscript{b}, John H. Henry\textsuperscript{c}, Ian A. Cunningham\textsuperscript{a,c}

\textsuperscript{a}Imaging Research Laboratories, Robarts Research Institute, The University of Western Ontario, 100 Perth Drive, London, Ontario, N6A 5K8, Canada; \textsuperscript{b}School of Mechanical Engineering, Pusan National University, Jangjeon-dong, Geumjeong-gu, Busan 609-735, Republic of Korea; \textsuperscript{c}London Health Sciences Centre, 339 Windermere Road, London, Ontario, N6A 5A5, Canada

ABSTRACT

A new method of measuring the zero-frequency value of the detective quantum efficiency (DQE) of x-ray detectors is described. The method is unique in that it uses what we call a “simulated neutral-attenuator” method to determine the system gain derived from image-based measurements of x-ray transmission through a thin copper foil of known thickness. Since this method uses only low-contrast image structure, it is a true measure of the “small-signal” system gain which is assumed piece-wise linear. A theoretical expression is derived for the linearized pixel value which is the pixel value that a linear system would have for the test conditions. Combining this with the measured detector exposure and zero-frequency value of the Wiener noise power spectrum provides the DQE. It is shown this method gives a DQE value that is in agreement with conventional test methods on a linear flat-panel detector, and that the same DQE value is obtained when using both raw (linear) and processed (non-linear) images.

Keywords: x-ray imaging, detective quantum efficiency, DQE, detector performance, quality assurance, cesium iodide

1. INTRODUCTION

The detective quantum efficiency (DQE) describes the effective quantum efficiency\textsuperscript{1} of an x-ray detector (in terms of signal-to-noise ratio capture) and is a surrogate measure of the detector “dose efficiency”\textsuperscript{2,3}. In addition, the DQE has fundamental importance for the description of radiographic imaging systems as it describes the performance of an ideal observer (under certain noise-limited conditions).\textsuperscript{4-6} and is therefore a good indicator of system performance. As with other Fourier-based metrics, it is valid only for linear and shift-invariant imaging systems and wide-sense stationary (WSS) noise processes.\textsuperscript{5-8}

The fundamental importance and quantitative nature of the DQE makes it attractive to use in routine quality assurance programs in radiological facilities. However, some manufacturers implement logarithmic or other non-linear system response characteristics and do not make linear image data available to the user. This imposes a serious practical limitation on the routine use of DQE in tests of detector performance.

We describe a method of measuring the zero-frequency value of the DQE that can be implemented using either raw (linear) or processed (non-linear) image data. In this context, linearity means the pixel value is proportional to x-ray intensity. The method is based on a “small-signal” approximation that assumes the system gain (ratio of average pixel value \( \bar{d} \) to area density of incident quanta, \( \bar{q} \) in quanta mm\(^{-2}\) is piecewise linear for small changes in \( \bar{q} \).
2. THEORY

The DQE can be described as the ratio of the Wiener noise-power spectrum (NPS) describing Poisson variations in the incident quanta, passed through an ideal (linear and noise-free gain) imaging system, to the measured image NPS for a system of the same gain and modulation transfer function (MTF): ²

\[
\text{DQE}(u) = \frac{W_q(u) G^2 T^2(u)}{W_d(u)}
\]

(1)

where \( W_q(u) \) [mm\(^{-2}\)] is the Wiener NPS associated with the incident x-ray distribution \( q \), \( u \) [mm\(^{-1}\)] is the spatial-frequency variable, \( T(u) \) is the system modulation transfer function (MTF), \( G = \bar{d}/\bar{q} \) is the system gain, and \( W_d(u) \) [mm\(^2\)] is the image NPS corresponding to \( \bar{q} \). For a Poisson distribution of incident x-ray quanta, \( W_q(u) = \bar{q} \).

If we restrict ourselves to evaluating the zero-frequency DQE value, a more general expression for the DQE that applies to both linear and non-linear systems can be written as:

\[
\text{DQE} = \frac{\bar{q} \left| \frac{\partial \bar{d}}{\partial \bar{q}} \right|^2}{W_d(0)}
\]

(2)

where the differential term is the ‘small-signal’ system gain. The small-signal approach has been used previously for non-linear systems such as film-screen technologies.⁹,¹⁰ Only for a true linear system does the small-signal gain equal the overall system gain, giving the familiar result:

\[
\text{DQE} = \frac{1}{\bar{q} W_d(0)/d^2}.
\]

(3)

The difficulty in applying Eq. (2) is how to determine the gain \( \partial \bar{d}/\partial \bar{q} \) which must be measured for the particular incident x-ray spectrum and exposure level desired. For example, while it may be possible to map \( \bar{d} \) as a function of \( \bar{q} \) by modifying the generator mAs setting and either linearize the system response or measure the slope at the desired exposure level, this is not possible on systems that may rescale image data depending on the exposure level, and hence is not a general solution. If it were possible, one could attenuate the beam with a ‘neutral’ attenuator for which the attenuation coefficient is independent of x-ray energy and determine the system response. Our approach is to adapt the neutral-attenuator concept to make use of an attenuator of specified material and thickness with a theoretical correction to compensate for spectral effects. This approach is restricted to use only standardized x-ray spectra for which the spectral shape is known, and we show has modest dependence on the detector composition.

2.1 Simulated Neutral-Attenuator Method

For an ideal linear detector, \( \bar{d} \) is proportional to the average x-ray energy deposited per unit area in the detector, \( \bar{E} \) [keV mm\(^{-2}\)]:

\[
\bar{d} = k \bar{E},
\]

(4)

where

\[
\bar{E} = \int_0^{kV} \bar{q}(E) \alpha(E) E_a(E) dE,
\]

\( k \) [mm\(^2\) keV\(^{-1}\)] is a constant of proportionality related to pixel size and other factors, \( \bar{q}(E) \) [mm\(^{-2}\)keV\(^{-1}\)] is the spectral distribution of incident x-ray quanta where \( \bar{q} = \int_0^{kV} \bar{q}(E) dE \) [mm\(^{-2}\)], \( \alpha(E) \) is the detector quantum efficiency, and \( E_a(E) \) [keV] is the x-ray energy deposited in the detector per interaction.

For non-linear images, \( \bar{d} = f(\bar{E}) \), and for the general case we use the chain rule of differentiation to describe the small-signal gain as:

\[
\frac{\partial \bar{d}}{\partial \bar{q}} = \frac{\partial \bar{d}}{\partial \bar{E}} \frac{\partial \bar{E}}{\partial \bar{q}}.
\]

(5)
The first term on the right-hand side is the rate of change in \( \hat{d} \) with respect to a change in deposited energy density \( \hat{E} \). The second term describes the change in \( \hat{E} \) with respect to a change in \( \bar{q} \) when the change in x-ray spectrum results from a scaling of the spectrum without changing the spectrum shape. In concept this could be achieved by attenuating the beam with a neutral attenuator made of a fictitious material having energy-independent linear-attenuation coefficient \( \mu_n \) and thickness \( t_n \):

\[
\frac{\partial \hat{E}}{\partial t} = \frac{\partial \hat{E}(t_n)}{\partial t_n} \times \left( \frac{\partial \bar{q}}{\partial t_n} \right)^{-1},
\]

where

\[
\frac{\partial \hat{E}(t_n)}{\partial t_n} = \frac{\partial}{\partial t_n} \left\{ \int_0^{kV} \bar{q}(E)\alpha(E)E_a(E)e^{-\mu_n t_n}dE \right\} = -\mu_n e^{-\mu_n t_n} \int_0^{kV} \bar{q}(E)\alpha(E)E_a(E)dE
\]

and

\[
\frac{\partial \bar{q}}{\partial t_n} = \frac{\partial}{\partial t_n} \left\{ \int_0^{kV} \bar{q}(E)e^{-\mu_n t_n}dE \right\} = -\mu_n e^{-\mu_n t_n} \bar{q}.
\]

Combining these results with Eq. 5 gives the small signal gain as:

\[
\frac{\partial \hat{d}}{\partial \bar{q}} = \frac{\partial \bar{d}}{\partial E} \hat{E}_a
\]

where \( \hat{E}_a = \int_0^{kV} s(E)\alpha(E)E_a(E)dE \) (with \( s(E) = \gamma(E)/\gamma \) is the average value of deposited energy \( E_a(E) \), weighted by the spectral distribution of interacting x-ray quanta.

At this point it is convenient to define \( d_e \) as an effective linear pixel value, equal to the pixel value that a linear system would produce if the gain was equal to the measured small-signal gain, defined as \( d_e = \frac{\partial \bar{d}}{\partial \bar{q}} = \frac{\partial \bar{d}}{\partial E} \hat{E} \). If we introduce a thin copper attenuator of attenuation coefficient \( \mu(E) \) and thickness \( t \) to cause a change in \( \hat{\bar{q}} \) and \( \hat{E} \), the effective linear pixel value is given by:

\[
d_e = \frac{\partial \hat{d}(t)}{\partial E(t)} \bigg|_{t=0} \times \hat{E} = \frac{\partial \hat{d}(t)}{\partial \hat{E}_a(t)} \bigg|_{t=0} \times \hat{E}_a
\]

where

\[
\hat{E}_a(t) = \int_0^{kV} s(E)\alpha(E)E_a(E)e^{-\mu(E)t}dE.
\]

With this,

\[
\text{DQE} = \frac{1}{\bar{q}W(0)/d_e^2}
\]

which is of the same form as the linear DQE by making use of the linearized pixel value \( d_e \).

An important strength of this form is that \( d_e \) depends on the shape of the spectrum and measured \( \hat{d} \) values, but not on the exposure level. At first glance it may appear from Eq. (11) that an accurate knowledge of \( \alpha(E) \) and \( E_a(E) \) is required to determine the DQE, but in fact the DQE is only weakly dependent on these terms. Another benefit of this form is that \( \bar{d}(t) vs. \hat{E}(t) \) will generally be well behaved, making a determination of the slope at \( t = 0 \) from one-sided data more robust. For example, it will be a near-straight line for a linear system and near-logarithmic for a log system.

3. EXPERIMENTAL VALIDATION

3.1 Validation of simulated neutral-attenuator method and \( d_e \)

The simulated neutral-attenuator method was validated experimentally on a CsI-based flat panel detector system (General Electric Revolution XRd QX/4) that had been validated as being linear over the entire exposure range used in the experiments. This system was chosen for this experiment as each image could be saved in both linear
(raw) and non-linear (processed, chest PA radiograph) modes. The processed images had an inverted gray scale and some edge enhancement. The average pixel value in each step was determined well away from the step edges to avoid edge effects caused by post processing. For this reason, it was not possible to use the narrow-beam method described below with processed images.

Using an RQA-5 spectrum (70 kV, 21 mm added Al, 7.1 mmAl HVL) and pieces of copper shim stock having nominal thicknesses of 0.0254, 0.0508, 0.1778, 0.3048 and 0.508 mm (as stated by the supplier), the pixel value \( \bar{d} \) was measured and the deposited energy \( E_d \) estimated from Eq. (11) for each. Copper mass attenuation coefficients were obtained from the NIST database using a copper density of 8.96 g/cm\(^3\). The detector quantum efficiency \( \alpha(E) \) was estimated as the probability of interaction assuming 0.471 g/cm\(^2\) of CsI. Three configurations were tested as summarized in Table 1.

The value of \( \partial \bar{d} / \partial E_d \) was determined by plotting measured values of \( \bar{d} \) as a function of \( E_d \) and fitting a second-order polynomial to the data. The effective pixel value \( d_e \) was calculated and compared with true pixel values measured from open field images using linear image data.

### 3.2 Validation of DQE

The system DQE was measured by conventional methods using “DQEPro” from DQE Instruments and is the standard to which the simulated neutral-attenuator method was compared. The frequency-dependent NPS was calculated from five open-field images and the value of \( \bar{q} \) determined using the IEC 6220-1 guideline from the measured incident exposure projected into the image plane using the inverse-square law.

### 4. RESULTS

A comparison of pixel value \( \bar{d} \) as determined for each copper thickness for each configuration and \( E_d \) values, all normalized to unity for \( t = 0 \), is shown in Fig. 1. The Cu-on-source data and narrow-beam data both closely follow the theoretical \( E \) curve. The wide-beam data shows slightly less attenuation for the same copper thickness. This is believed to be due to detector glare, seen as an 8% low-frequency drop in the MTF, which adds an approximately uniform offset to image data.

Figure 2 shows \( \bar{d} \) as a function of \( E \) for narrow and wide beam data from linear images. As expected, the curve is an approximately straight line for the narrow-beam data. The wide-beam data deviates slightly from a straight line as the copper thickness is increased (lower \( E \) values) as the effect of detector glare becomes more pronounced. However, the slope corresponding to \( t = 0 \) (maximum \( E \) value), and therefore the \( d_e \) value, is remarkably insensitive to glare. Table 2 confirms that values of \( \bar{d} \) and \( d_e \) are approximately equal for the linear image data, as expected. The difference is 1 - 2% for narrow-beam data and 5% for wide-beam data. Figure 2 also shows the corresponding result for the wide beam using processed images. As expected, the slope is negative since image contrast is inverted, and the curve is not a straight line.

Evaluation of the zero-frequency NPS value is the most difficult part of this analysis. Figure 3 shows the NPS for both linear and non-linear images. The linear-image NPS shows a smooth transition to the zero-frequency axis while the processed-image NPS peaks at a low frequency value. The zero-frequency NPS value was estimated to be 14.2 and 24.5 mm\(^2\) for the raw and processed images respectively.

<table>
<thead>
<tr>
<th>Cu Distance from Detector</th>
<th>Beam Width on Detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu on Source</td>
<td>100 cm</td>
</tr>
<tr>
<td>Narrow Beam</td>
<td>20 cm</td>
</tr>
<tr>
<td>Wide Beam</td>
<td>20 cm</td>
</tr>
</tbody>
</table>

Table 1. Beam size on detector and distance of copper attenuator from detector for each test configuration.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( d_e ) from Eq. 10</th>
<th>( d ) from linear image data</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu on Source (no scatter from Cu)</td>
<td>878</td>
<td>866</td>
<td>1.4%</td>
</tr>
<tr>
<td>Narrow Beam</td>
<td>788</td>
<td>773</td>
<td>2.0%</td>
</tr>
<tr>
<td>Wide Beam</td>
<td>1267</td>
<td>1336</td>
<td>-5.2%</td>
</tr>
</tbody>
</table>

Table 2. Comparison of effective pixel values with true pixel values obtained using linear image data.
Figure 1. Comparison of theoretical $\hat{E}$ curve with $\bar{d}$ values determined from narrow-beam, wide-beam, and Cu-on-source using linear image data. The wide-beam data does not decrease as quickly, believed to be due to detector glare which adds an approximately uniform (DC) offset to pixel values (and a low-frequency drop to the MTF). Each curve is normalized to unity at $t = 0$ for comparison.

Figure 2. Plot of $\bar{d}$ vs. $\hat{E}$ from linear (left) and non-linear (right) image data. The slope at the far right-hand point for each curve, $\partial d / \partial E$, was determined from a second-order polynomial fit through each line as illustrated. Effective pixel values determined from this data are summarized in Table 2.
Figure 3. The NPS shown here was determined from both raw (left) and processed (right) image data. The zero-frequency value was estimated by extrapolating the NPS to the zero-frequency axis, ignoring the low-frequency increase seen in the processed image data result.

Figure 4. The zero-frequency DQE value obtained using the neutral-attenuator method with both raw (linear) and processed (non-linear) images is compared with a conventional calculation using linear images. The conventional analysis gives a zero-frequency value of 0.50 while the neutral-attenuator method gives 0.49 and 0.47 using linear and non-linear images respectively.

The DQE obtained with Eq. (12) is shown in Fig. 4 and compared with a conventional frequency-dependent DQE analysis. Excellent agreement was obtained, however the accuracy of the method is not yet established and is dependent on both the accuracy of determining $d_e$ and the NPS zero-frequency value.

The simulated neutral attenuator method requires knowledge of the detector material. For example, Table 3 shows that underestimating the detector material thickness leads to reduced $d_e$ values. However, this dependence is very modest for the CsI detector. For example, the true CsI thickness is likely between 0.5 mm (0.25 g/cm²) and 1.0 mm (0.5 g/cm²), and the $d_e$ value varies by only 1% over this range. In fact, if one errs by assuming a quantum efficiency of unity, the error is only 2%. Thus, it appears that if detector specifics are not known, one should make this assumption and then even the detector material is not important.
5. CONCLUSIONS

A new method has been developed to measure the zero-frequency DQE value that can be applied to both linear and non-linear image data. The method makes use of copper attenuators with a robust theoretical correction to simulate a neutral attenuator for which the attenuation coefficient is independent of energy. The method was validated on a CsI-based α-Si digital flat panel detector. A DQE value of approximately 0.5 was obtained using both linear and non-linear image data, and agreed with a conventional assessment using linear images.

<table>
<thead>
<tr>
<th></th>
<th>Truth</th>
<th>0.1 g/cm²</th>
<th>0.25 g/cm²</th>
<th>0.5 g/cm²</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>CsI</td>
<td>866</td>
<td>811 (-6%)</td>
<td>838 (-1%)</td>
<td>866 (0%)</td>
<td>883 (2%)</td>
</tr>
<tr>
<td>Se</td>
<td>866</td>
<td>736 (-15%)</td>
<td>783 (-10%)</td>
<td>827 (-5%)</td>
<td>883 (2%)</td>
</tr>
</tbody>
</table>

Table 3. Comparison of $d_e$ with true $d$ from linear image data for a range of detector materials and thicknesses (g/cm²). The detector material is CsI. While the true detector material thickness is not known, it is almost certainly between 0.25 and 0.5 g/cm² (0.5 and 1.0 mm).

REFERENCES