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Monte Carlo analysis of megavoltage x-ray interaction-induced signal and noise in cadmium tungstate detectors for cargo container inspection

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ABSTRACT: For the purpose of designing an x-ray detector system for cargo container inspection, we have investigated the energy-absorption signal and noise in CdWO₄ detectors for megavoltage x-ray photons. We describe the signal and noise measures, such as quantum efficiency, average energy absorption, Swank noise factor, and detective quantum efficiency (DQE), in terms of energy moments of absorbed energy distributions (AEDs) in a detector. The AED is determined by using a Monte Carlo simulation. The results show that the signal-related measures increase with detector thickness. However, the improvement of Swank noise factor with increasing thickness is weak, and this energy-absorption noise characteristic dominates the DQE performance. The energy-absorption noise mainly limits the signal-to-noise performance of CdWO₄ detectors operated at megavoltage x-ray beam.

KEYWORDS: Detection of contraband and drugs; Detector modelling and simulations I (interaction of radiation with matter, interaction of photons with matter, interaction of hadrons with matter, etc); Inspection with x-rays

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1 Introduction

Related to homeland security and contraband control, an x-ray scanning system can increase the effectiveness and efficiency of cargo screening at the port [1]. To reduce radiopacity for heavy cargo, megavoltage (MV) x-ray beam generated by a linear accelerator (LINAC) is typically used as a probe for cargo inspection. To scan the large field-of-view of cargo container, linear arrays of scintillation crystals (as a detector), which convert incident x-ray photons to optical photons, and an LINAC (as a source) are on opposite sides of a container to be scanned and the container slowly moves in the perpendicular direction of the MV x-ray beam. To avoid radiation damage, the photodiode arrays, which detect the optical photons emitted from the scintillator arrays, are placed to the sides of detector arrays, as schematically illustrated in figure 1(a). As a scintillation detector, a CdWO$_4$ crystal is a good candidate [2] because it shows reasonable properties required for this specific application with respect to density [3], effective atomic number [3], radiation damage [4], and optical transparency [5], although it suffers from low light yield, poor matching with the sensitivity curve of the photodiode, and toxicity [3].

While high quality x-ray images, from which one identifies abnormal objects visually and quantitatively, are demanded, MV images are normally expected to suffer from poor quality due to high energy-absorption noise [6, 7]. At MeV energies, the photoelectric events are less probable whereas Compton-scatter events are more, as shown in figure 1(b). Escapes of scattered photons or annihilation photons, as a result of pair production, from a detector will increase energy-absorption noise. Detectors can also miss energetic electrons produced by each photon interaction, and which further increases energy-absorption noise. Therefore, the design of detector system is undoubtedly crucial considering MV x-ray interaction-induced signal-to-noise performance, which will be the upper
limit of ultimate signal-to-noise performance of the detector system because further conversion processes into light and electronic quanta increase noise disproportionately to signal enhancement.

In this work, we investigate the signal-to-noise performance of CdWO$_4$ detectors having various thicknesses at MeV x-ray energies by using a Monte Carlo (MC) technique. As a performance descriptor, we employ the detective quantum efficiency (DQE) that describes a measure of detector performance to capture the signal-to-noise ratio of the incident x-ray photon fluence [8]. Energy-absorption noise itself can be assessed by the Swank noise factor [9], which is implicitly included in DQE [10] [see eq. (2.3) below]. Higher Swank factor implies less absorption noise. Those DQE and Swank factor can be expressed in terms of energy moments of the absorbed energy distributions (AEDs) [6, 9–11] and we determine the AEDs of CdWO$_4$ detectors using the MC simulations. The impact of electron transport in the MC simulations on the AED and the signal and noise performance are investigated by separately performing MC simulations suppressing the electron transport. The effect of spectral distribution of incident MV-photon energies on the signal and noise performance is also assessed by comparing the simulation results to those obtained for the monoenergetic x-ray beam whose energy corresponds to the average energy of the MV spectrum. This work will suggest fundamental limits of the signal-to-noise performance of CdWO$_4$ detectors due to MV x-ray interactions, and the results will be useful for the detector design of container-inspection detector systems.

2 Theoretical background

2.1 Energy absorption-induced signal and noise

We are interested in the energy absorption-induced signal-to-noise characteristics of a CdWO$_4$ detector, and figure 2(a) shows a simple linear cascade model describing the signal and noise transfer in a CdWO$_4$ detector. $\tilde{q}_0$ represents a random variable describing the number of incident
Figure 2. (a) A simple cascade model describing the quantum signal and noise transfer in a CdWO₄ detector system for incidence of x-ray photons ($\tilde{q}_0$): photon detection ($\tilde{\alpha}$), energy deposition ($\tilde{\beta}$), and energy-to-output signal conversion ($k$). This model excludes any additional noise (e.g., electronic noise). The overhead tilde designates a random variable. (b) A rectangular parallelepiped with dimensions of $x$ (height) $\times$ $y$ (width) $\times$ $z$ (thickness) to model the CdWO₄ detector geometry for the Monte Carlo simulations.

x-ray photons. The model consists of the following stages: (1) ‘binomial’ x-ray photon detection with mean ($\bar{\alpha}$) and variance ($\bar{\alpha} - \bar{\alpha}^2$), (2) energy absorption per x-ray detection with mean ($\bar{\beta}$) and variance ($\bar{\beta}^2 - \bar{\beta}^2$), and (3) deterministic conversion of energy-to-output signal ($k$). Then, the output signal is given by

$$\bar{d} = k\tilde{q}_0\bar{\alpha}\bar{\beta}. \quad (2.1)$$

On the other hand, since output noise of an arbitrary gain stage with mean ($\bar{g}$) and variance ($\sigma^2_g$) for an incidence of $\tilde{q}_\text{in}$ is $\sigma^2_\text{out} = \bar{g}^2\sigma^2_\text{in} + \sigma^2_g\tilde{q}_\text{in}$, the output signal variance of the model is given by

$$\sigma^2_d = k^2\tilde{q}_0\bar{\alpha}^2. \quad (2.2)$$

Assuming that $\tilde{q}_0$ follows the Poisson statistics and using eqs. (2.1) and (2.2), we obtain

$$\text{DQE} = \frac{\bar{d}^2}{\tilde{q}_0} = \frac{\bar{\alpha}^2\bar{\beta}^2}{\bar{\beta}^2} = \bar{\alpha}I_A, \quad (2.3)$$

where $I_A$ is the Swank noise factor.

2.2 Moment analysis of AED

In this subsection, we connect the cascade model parameters shown above and the AED, that will be determined by the MC simulations in this study, using the concept of energy moments. The AED in a detector represents the probability that a photon with energy $E$ interacts in the detector and deposits an energy $\epsilon$ [12]. We let $A(\epsilon, E)$ represent the AED for an incident photon having energy $E$:

$$A(\epsilon, E) = \alpha(E)R(\epsilon, E), \quad (2.4)$$

where $\alpha(E)$ is the energy-dependent quantum efficiency and $R(\epsilon, E)$ is the detector response function describing the probability density of depositing energy $\epsilon$ given an interacting x-ray photon with energy $E$. The $n$th energy moment of the incident-energy-dependent AED is then given by

$$M_n(E) = \int_0^\infty A(\epsilon, E)e^n\epsilon d\epsilon. \quad (2.5)$$
In the case of a broad spectrum of x-ray photons, $S(E)$, eq. (2.5) can be averaged over the normalized spectrum $s(E) = S(E) / \int_{0}^{\infty} S(E)dE$:

$$M_n = \int_{0}^{\infty} s(E)M_n(E)dE = \int_{0}^{\infty} A(\epsilon)\epsilon^n d\epsilon,$$

(2.6)

where

$$A(\epsilon) = \int_{0}^{\infty} s(E)A(\epsilon, E) dE.$$

(2.7)

To calculate an energy moment, we therefore need to know $A(\epsilon)$ for $s(E)$.

Substituting eq. (2.7) into eq. (2.6), the zeroth moment yields

$$M_0 = \int_{0}^{\infty} s(E)A(E)dE = \bar{\alpha}.$$

(2.8)

Similarly, the first and second moments are

$$M_1 = \int_{0}^{\infty} s(E)A(E)\beta(E)dE = \bar{\alpha}\bar{\beta},$$

(2.9)

and

$$M_2 = \int_{0}^{\infty} s(E)A(E)\beta^2(E)dE = \bar{\alpha}\bar{\beta}^2,$$

(2.10)

respectively, where $\beta^n(E)$ is the $n$th moment of $R(\epsilon, E)$. Therefore, the DQE can be expressed in terms of energy moments:

$$\text{DQE} = \frac{M_2^2}{M_2}$$

(2.11)

with

$$I_A = \frac{M_1^2}{M_0M_2}.$$

(2.12)

3 Methods

3.1 Monte Carlo simulations

Based on the theoretical developments above, we investigate the signal-to-noise performance of CdWO$_4$ detectors. To determine $A(\epsilon)$, we carried out virtual pulse-height spectroscopy measurements using the MCNP code (version 5, RSICC, Oak Ridge, TN) to simulate the coupled photon-electron transport within a detector [13]. X-ray photon energies were sampled from a 9-MV spectrum, as shown in figure 1(b), which was obtained from the Varian Medical Systems, Inc. (Security & Inspection Products, Las Vegas, NV). The average energy of the spectrum was $\bar{E}_s \approx 0.8$ MeV. The CdWO$_4$ detector with a density of 7.9 g cm$^{-3}$ was simply modeled as a rectangular parallelepiped with dimensions of $x$ (height) $\times$ $y$ (width) $\times$ $z$ (thickness), as shown in figure 2(b). The $y-z$ plane corresponds to a photodiode area. While $x$ and $y$ were fixed at 4 mm, $z$ was varied from 10 mm to 50 mm. In all cases, a pencil-like photon beam was perpendicularly incident upon the center point of the $x-y$ plane of the detector. For comparison, we also determined $A(\epsilon)$ for a monoenergetic photon beam with energy $\bar{E}_s$. 
The electron transport is optional in the MCNP code. The electron-transport option can account for energy loss from a detector due to escapes of high-energy Compton-recoiled electrons, photoelectrons, or pair-production products in the simulations, whereas it significantly retards the MC running time. We compare the MC simulation results obtained when the electron-transport option is turned ‘on’ and ‘off’, and report them by calculating the relative difference ($\delta$) with respect to the ‘on’ cases (in units of %). The MC simulations included $10^8$ incident photon histories with energy-absorbing events summed in 0.01-MeV energy bins.

### 3.2 Verification

We compared the MC results with a simple analytic approach using the XCOM photon cross-section library (NIST, MD). In this case the AED is given by [13]

$$A(E) = S(E) \left(1 - e^{-\mu_{\text{tot}}(E)z}\right) \frac{\mu_{\text{en}}(E)}{\mu_{\text{tot}}(E)},$$

(3.1)

and $\bar{\alpha}$ and $\bar{\beta}$ are given by

$$\bar{\alpha} = \frac{\int_{0}^{\infty} S(E) \left(1 - e^{-\mu_{\text{tot}}(E)z}\right) dE}{\int_{0}^{\infty} S(E) dE},$$

(3.2)

and

$$\bar{\beta} = \frac{\int_{0}^{\infty} S(E) \left(1 - e^{-\mu_{\text{tot}}(E)z}\right) \frac{\mu_{\text{en}}(E)}{\mu_{\text{tot}}(E)} E dE}{\int_{0}^{\infty} S(E) \left(1 - e^{-\mu_{\text{tot}}(E)z}\right) dE},$$

(3.3)

where $\mu_{\text{tot}}$ and $\mu_{\text{en}}$ denote total linear attenuation coefficient and energy-absorption coefficient, respectively.

While the Swank factor $I_A$, as shown in eq. (2.12), includes the statistical uncertainty of incident spectral photon energies and variation in energy absorption within a detector, the Swank factor obtained for a monoenergetic beam, $I(\bar{E}_s)$, will include only variation in energy absorption due to monoenergetic photon interactions. We calculated the spectrum uncertainty $I_s$ as 0.41 for the 9-MV spectrum and compared $I_s \times I(\bar{E}_s)$ and $I_A$. Similarly, we compared $I_s \times \text{DQE}(\bar{E}_s)$ and DQE.

### 4 Results

Figure 3 compares the AEDs of CdWO$_4$ detectors with thicknesses of 10 mm and 50 mm obtained from MC simulations for various combinations of simulation plans. The AEDs for the monoenergetic beam show large photopeaks at $\epsilon = \bar{E}_s$, and escapes peaks at $\epsilon = \bar{E}_s - 26.7$ keV and $\epsilon = \bar{E}_s - 69.5$ keV due to Cd and W, respectively. The AEDs for the 9-MV spectrum follow generally the shape of $s(E)$ but show no escape peaks. The electron transport in the simulations underestimates slightly the photon-only simulation results for energy bins greater than 4 MeV, as shown in figure 3(b), but the differences are negligible. It is expected high Swank factors for the monoenergetic-beam simulations because of dominant photopeaks, whereas low Swank factors for the 9-MV spectral simulations because of broad AEDs.

As shown in figure 4(a), the monoenergetic simulation underestimates the quantum efficiency obtained from the spectral simulation, and this discrepancy is reduced with increasing detector thickness. The reason is because at 0.8 MeV the quantum efficiency is mostly described by the
Figure 3. $A(\epsilon)$ curves for the (a) monoenergetic beam and (b) 9-MV spectrum obtained from Monte Carlo simulations for CdWO$_4$ detectors with $z = 10$ mm and 50 mm. Each figure compares the photon-only and photon-electron coupled simulations. For brevity, only the results obtained from photon-electron coupled simulations include error bars.

Compton scattering (> 80%), as shown in figure 1(b). The analytic calculations using eq. (3.2) well describe both the MC results.

On the other hand, the monoenergetic simulation overestimates largely the average energy absorption per x-ray interaction compared to the spectral simulation, as shown in figure 4(b). The monoenergetic simulation result is independent of detector thickness because monoenergetic x-ray photons result in an almost constant fractional energy absorption per interaction. Since $\mu_{en}$ does not include reabsorption of scattered-photon energies (it only accounts for radiative loss), the analytic estimation based on eq. (3.3) underestimates the monoenergetic simulation result. Average energy absorption per incident x-ray photon or $\bar{\alpha} \bar{\beta}$ as a function of detector thickness is presented in figure 4(c). The agreement between the spectral simulation and the analytic calculation is excellent.

The Swank noise factor for the 9-MV spectrum is about 0.3, as shown in figure 4(d), for all the detector thicknesses under investigation in this study. As expected from the AED analysis, the monoenergetic simulation overestimates largely the Swank noise factor by a factor of ~ 2.7 compared to the spectral simulation. Further consideration of the Swank noise factor due to uncertainty in the spectrum itself into the monoenergetic simulation result reduces greatly the discrepancy with the spectral simulation result. This observation shows that the broadness of incident photon spectral distribution can affect largely the energy-absorption noise of a detector. The Swank noise factor is nearly independent of detector thickness.

As shown in figure 5(a), the magnitude of DQE is mainly determined by the Swank noise factor. Monoenergetic simulation overestimates largely the DQE performance. However, the monoenergetic simulation (or analytical approach) with a separate consideration of the spectrum Swank factor can describe reasonably the spectral simulation result. This is an important finding of this study.

Figure 5(b) summarizes the relative difference between the analyses with and without the electron transports. Although the Swank noise factor is mostly affected by the electron transport, the relative difference is less than 3%. In this work, the x-ray pencil beam was incident on the
Figure 4. (a) Quantum efficiency. (b) Average energy absorption per interacting x-ray photon. (c) Average energy absorption per incident x-ray photon. (d) Swank noise factor. The lines in (a), (b), and (c) describe the analytical calculation results, whereas the line in (d) describes the corrected result of $I(E_s)$ with $I_s$.

Figure 5. (a) DQE. The line describes the corrected result of DQE($E_s$) with $I_s$. (b) Relative difference between the energy-moment analysis results based on Monte Carlo simulations with and without electron transport options.

center of the detector, hence escapes of high-energy scattered photons and electrons through the four sides of the detector might be negligible. This situation would not be true in practice. The
effects of incident beam locations on AED and Swank noise factor should be investigated.

5 Discussion

In the cascaded-systems analysis, we assumed that the signal and noise transferring stages after the energy-absorption stage ($\beta$) were deterministic and treated them as a single gain factor of $k$ without its stochastic variance. However, this is not true. The $\beta$ stage can be followed by the production of light photons, the collection of light photons at the photodiode, the conversion of light photons into electrons, the aperture integration (over the $y-z$ plane) of electronic signal, and the addition of additive noise (e.g., thermal noise electronic quanta), and all these processes are stochastic [14]. Therefore, further increase in detector noise is obvious. In this study, however, we are interested in the signal-to-noise performance induced by only energy absorption in the CdWO$_4$ detector, and which will be the upper limit of the detector system that we can achieve. More detailed cascaded-systems analysis will be our separate study in the near future.

This study showed that the DQE of a CdWO$_4$ detector increased as its thickness increased. Again, this study only accounted for the signal and noise due to energy absorption and ignored those due to the light photon interactions and their electronic conversions including the additive electronic noise terms. For example, since thermal noise of photodiode is in general proportional to the square root of the photodiode area [15], $\sigma_{\text{thermal}} \propto \sqrt{J A_{\text{PD}}}$, where $J$ and $A_{\text{PD}}$ denote the thermal current density and the photodiode area, respectively, it would not be desirable to design a CdWO$_4$ detector with a thickness that gives the highest DQE performance without considering additional noise terms. Rather, there would be a compromise in detector thickness. The cascaded-systems analysis taking into account the additional terms as described above will be useful for determining the optimum detector thickness.

6 Conclusion

We have investigated the upper limit of signal-to-noise performance in CdWO$_4$ detectors having various thicknesses for 9-MV container inspection system applications. While increasing detector thickness enhances photon detection and energy deposition, the thickness effect on energy-absorption noise is less significant and this absorption noise dominates the signal-to-noise performance or DQE. The energy-absorption noise would be a most significant bottleneck in the design or selection of detector material at MeV energies. Since the DQE gradually increases with detector thickness and then almost saturates at the thickness of 30 mm, this study suggests the minimum CdWO$_4$ thickness would be 30 mm for a 9-MV container inspection system in terms of energy-absorption signal and noise. An important secondary finding of this study is that a simple analytic approach based on the Lambert-Beers law can describe reasonably MV x-ray interaction-induced signal and noise performance.

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