Spectral analysis of fundamental signal and noise performances in photoconductors for mammography

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Purpose: This study investigates the fundamental signal and noise performance limitations imposed by the stochastic nature of x-ray interactions in selected photoconductor materials, such as Si, α-Se, CdZnTe, Hgl2, PbI2, PbO, and TlBr, for x-ray spectra typically used in mammography.

Methods: It is shown how Monte Carlo simulations can be combined with a cascaded model to determine the absorbed energy distribution for each combination of photoconductor and x-ray spectrum. The model is used to determine the quantum efficiency, mean energy absorption per interaction, Swank noise factor, secondary quantum noise, and zero-frequency detective quantum efficiency (DQE).

Results: The quantum efficiency of materials with higher atomic number and density demonstrates a larger dependence on convertor thickness than those with lower atomic number and density with the exception of α-Se. The mean deposited energy increases with increasing average energy of the incident x-ray spectrum. Hgl2, PbI2, and CdZnTe demonstrate the largest increase in deposited energy with increasing mass loading and α-Se and Si the smallest. The best DQE performances are achieved with PbO and TlBr. For mass loading greater than 100 mg cm⁻², α-Se, Hgl2, and PbI2 provide similar DQE values to PbO and TlBr.

Conclusions: The quantum absorption efficiency, average deposited energy per interacting x-ray, Swank noise factor, and detective quantum efficiency are tabulated by means of graphs which may help with the design and selection of materials for photoconductor-based mammography detectors. Neglecting the electrical characteristics of photoconductor materials and taking into account only x-ray interactions, it is concluded that PbO shows the strongest signal-to-noise ratio performance of the materials investigated in this study. © 2012 American Association of Physicists in Medicine. [http://dx.doi.org/10.1118/1.3702455]

Key words: cascaded-systems analysis, detective quantum efficiency, mammography, Monte Carlo, photoconductor, quantum efficiency, Swank noise

I. INTRODUCTION

Breast cancer is the most common cancer in women,¹ affecting one in nine women in North America. Early detection by mammography is challenging due to the need to visualize low-contrast lesions and microcalcifications. Analog mammography systems use film-screen technology to detect transmitted x-rays as well as record and display image information. Digital mammography, often using large-area semiconductor-based detectors, is able to address limitations of film-screen systems for imaging dense breasts,² as demonstrated in a study by Pisano et al.,³ which is particularly important when screening younger populations where the benefits of screening mammography remain controversial.⁴,⁵ Despite much progress in the development of alternative systems,⁶ such as dedicated breast computed tomography, digital tomosynthesis and magnetic resonance imaging, two-dimensional (2D) mammography remains the clinical standard for breast cancer screening.⁷
In any x-ray imaging task, the detector must provide images of high diagnostic quality while maintaining a safe level of patient x-ray exposure. This balance is best described by the detective quantum efficiency (DQE) of the detector.\(^8\)\(^-\)\(^10\) In mammography, high spatial resolution and low detector electronic noise are also critical.\(^11\) However, even if detector noise is negligible, the stochastic nature of quantum-based imaging systems remains a source of image noise and reduced DQE.\(^12\) This is particularly true for high-resolution imaging where a secondary quantum sink may exist at high spatial frequencies if the number of secondary quanta contributing to final image signal per interacting x-ray photon is much less than approximately 100.\(^10\),\(^12\) In general, both 2D and three-dimensional x-ray imaging techniques are optimized only with appropriate selection of high-gain x-ray converter materials.

Direct-conversion x-ray imaging methods, which use a photoconductive material to convert incident x-ray quanta directly into image forming quanta, have better spatial resolution characteristics than indirect-conversion methods that use optical scintillators.\(^13\)\(^-\)\(^17\) However, important drawbacks of these materials have been identified,\(^17\),\(^22\)\(^-\)\(^27\) although limitations imposed by electrical properties, such as mean drift length, leakage current, material stability, reproducibility, and large-area availability hamper their development.\(^17\)\(^-\)\(^21\)

Optimal image quality can be achieved only when the DQE is as close to unity as possible. For example, x-ray and optical scatter in a scintillator impact on both spatial resolution and image noise,\(^30\)\(^-\)\(^32\) resulting in a degraded DQE. Sakellaris et al. used Monte Carlo methods to investigate x-ray interaction physics in a-Se and their effect on spatial resolution,\(^33\) and extended their study to a wide range of photoconductor materials.\(^34\) Hajdok et al.\(^35\) investigated the DQE and x-ray Swank noise term in Si, a-Se, CsI, and PbI\(_2\) for a wide range of monoenergetic incident photons. The energy width of x-ray spectra also has a negative impact on the DQE.\(^36\)

Cascaded-systems analysis (CSA) is an effective tool for describing how the physical processes that contribute to the final image affect the DQE and image quality. Since CSA is Fourier based, it applies to systems that are linear and shift-invariant and involve only wide-sense stationary (WSS) or wide-sense cyclostationary noise processes.\(^10\) Monte Carlo (MC) calculations are sometimes required to account for complex physical process or geometries. For example, at megavoltage (MV) energies, multiple scattering events can be difficult to represent using CSA.

Some investigators have used MC methods to calculate absorbed energy distributions (AED) and then used CSA methods to examine the effect of blur and other factors on image quality and the DQE.\(^37\)\(^-\)\(^39\) While this approach has been used widely, it remains an approximation that does not correctly take into consideration the statistical nature of secondary quanta (electron-hole pairs or optical quanta) generation and collection, and therefore ignores a possible degradation in the DQE due to a secondary quantum sink. At MV energies, this is generally a reasonable approximation due to the very large number of secondary quanta generated for each interacting x-ray photon. At the lower mammographic energies, however, statistics of secondary quanta generation and collection efficiencies must be carefully considered as part of the design process. There are several commercial phosphor-based detectors in current use that suffer from a secondary quantum sink due to this problem. The requirement for high conversion gain will likely become more stringent with the development of photon-counting technologies.

In this paper, we introduce a method of combining the best of each approach that incorporates MC methods to determine the AED with a CSA model to calculate the DQE, and discuss optimal photoconductors for digital mammography based on the simulation results. Other considerations, including grain boundary problems inherent in polycrystalline materials such as PbI\(_2\) and HgI\(_2\) which may affect resolution and charge collection,\(^22\) are not specifically addressed.

II. THEORY

When an x-ray photon is incident on a detector, both the probability of interaction and the energy deposited (and therefore, the average number of secondary quanta generated) are functions of the energy of the incident photon. This violates a requirement of the CSA approach that each process in the cascade of processes be statistically independent. In other words, since the photon energy determines both the probability of interaction and the conversion gain, we cannot represent these as two different cascaded processes. Our solution, inspired in part by Rabbani and Van Metter’s description of an input-labeled random amplification process,\(^40\) is to represent both the selection of interacting quanta and subsequent conversion to secondary quanta as a single process with the additional feature that both the probability of interaction and the mean conversion gain are functions of a random variable representing the energy of each incident photon. We call this new process a “photon-interaction” process. The input is a spatial point distribution of incident quanta and the output is a spatial point distribution of secondary quanta at the locations where they are liberated within the photoconductor.

II.A. Photon-interaction process

In this process, selection of interacting photons is described as multiplication by the Bernoulli random variable \(z_n(E'_n)\) which can have sample values 1 or 0 only, and where \(E'_n\) is a random variable describing the photon energy where \(n\) identifies the \(n\)th incident photon. At each energy, this represents a gain stage where the gain mean and variance are given by \(z(E')\) and \(z(E') - z^2(E')\), respectively.\(^10\) Each interacting x-ray photon subsequently produces \(g_n(E'_n)\) spinoff.
secondary quanta with mean and variance \( \bar{g}(E') \) and \( \sigma_g^2(E') \), respectively. Propagation of the mean and Wiener noise power spectrum (NPS) through this new photon-interaction stage are developed in Appendix. For the special case of a WSS input, the mean of \( q_{\text{out}}(r) \) takes the form

\[
q_{\text{out}} = \langle xg \rangle \bar{q}_{\text{in}},
\]

where \( \langle \cdot \rangle \) indicates an average over energy-dependent terms weighted by the incident x-ray spectrum \( s(E') \). The NPS takes the form

\[
W_{\text{out}}(u) = \langle xg \rangle^2 |W_{\text{in}}(u) - \bar{q}_{\text{in}}| + \langle x\sigma_g^2 + x\sigma_q^2 \rangle \bar{q}_{\text{in}}.
\]

Equations (1) and (2) are new contributions and describe the transfer of mean signal and noise, respectively, through the photon-interaction process. In terms of mathematical notation, the overhead tilde is used to indicate a random variable, and the boldface \( r \) and \( u \) indicate vector-form variables in spatial and spatial-frequency domains, respectively.

II.B. Cascaded model of photoconductor detector

Starting with the above photon-interaction process, we describe each photoconductor detector using a CSA model consisting of a cascade of five linear stages as shown in Fig. 1. The input to the first stage is a distribution of x-ray quanta with expected value \( \bar{q}_0 \) (quanta per square millimeter) and NPS \( W_0(u) = \bar{q}_0 \).

II.B.1. X-ray interaction stage

This stage describes x-ray interaction and conversion to secondary quanta within the photoconductor. Using the transfer relationships described above, we obtain

\[
\bar{q}_1 = \langle xg \rangle \bar{q}_0,
\]

in per square millimeter and

\[
W_1(u) = \langle xg \rangle^2 |W_0(u) - \bar{q}_0| + \langle x\sigma_g^2 + x\sigma_q^2 \rangle \bar{q}_0
= \langle x\sigma_g^2 + x\sigma_q^2 \rangle \bar{q}_0,
\]

in per square millimeter. Here, the term \( \sigma_g^2 \) accounts for variability in the number of charge carriers generated at a particular x-ray energy and \( \sigma_q^2 \) accounts for secondary quantum noise due to the finite number of secondary quanta. For a monoenergetic beam, they are equivalent to a conventional cascade of selection and quantum gain processes. The above expression can be rewritten as

\[
W_1(u) = \langle x\sigma_g^2 \rangle \bar{q}_0,
\]

using the relation \( \sigma_g^2 = g^2 - \bar{g}^2 \).

Equations (3) and (5) are particularly important because they identify what information must be provided by the Monte Carlo analysis for incorporation into a cascaded model, and we show here that this comes from moments of the AED. This is seen by letting \( A(E,E') = \bar{z}(E)R(E,E') \) represent the probability that an incident photon of energy \( E \) deposits energy \( E \) in the photoconductor where \( R(E,E') \) is the probability that an interacting photon of energy \( E' \) deposits its energy \( E \). The AED for a spectrum of x-ray energies \( A(E) \) is obtained by averaging \( A(E,E) \) over \( s(E') \) as schematically illustrated in Fig. 2. The \( j \)th moment of \( A(E,E') \) for incident energy \( E' \)

\[
M_j(E') = \int_0^\infty E'\bar{z}(E')R(E,E')dE = \bar{z}(E')E(E'),
\]

where \( E(E') \) is the mean deposited energy per interacting photon. Averaging over \( s(E') \) gives moments of the AED for a spectrum of incident photons

\[
M_j = \int_0^\infty s(E')M_j(E')dE' = \bar{z}(E')g(E'),
\]

Assuming that \( g = \bar{E}/w \) and \( \bar{g}^2 = \bar{E}^2/w^2 \) where \( w \) is the effective energy required to liberate one electron-hole pair in the photoconductor gives

\[
\bar{q}_1 = \frac{M_1\bar{q}_0}{w} \quad \text{and} \quad W_1(u) = \frac{M_2\bar{q}_0}{w^2}.
\]

The above expressions are also new contributions. They provide a rigorous description of how MC calculations of the AED can be incorporated into a cascaded model and identify the requisite assumptions.

II.B.2. Collection of charge carriers by collecting electrodes

It is assumed that on average a fraction \( \beta \) of all liberated charge carriers will be collected at the electrodes. Factors such as electron-hole recombination and charge trapping will result in a decrease in \( \beta \). While in general \( \beta \) may depend on the depth of interaction, \( \delta \) for simplicity, we ignore this dependence. This process can be represented as a quantum selection stage, giving

\[
\bar{q}_2 = \beta \bar{q}_1 = \frac{\beta M_1\bar{q}_0}{w},
\]

in per square millimeter and

\[
W_2(u) = \beta^2W_1(u) + \sigma_q^2\bar{q}_1 = \beta^2\bar{q}_0 \left[ \frac{M_2}{w^2} + \frac{1 - \beta M_1}{\beta w} \right],
\]

in per square millimeter. This result shows how secondary quantum noise and potentially a secondary quantum sink caused by poor collection efficiency affect the NPS.

II.B.3. Random relocation of secondary quanta

Electron-hole pairs liberated in the photoconductor will undergo a number of random interactions and charge diffusion under the applied electric field, resulting in random relocation of each secondary quantum in the image plane prior to being collected at the collecting electrodes. We
assume all secondary quanta are relocated independently and with a probability described by the same point-spread function, giving:

$$\hat{q}_3 = \hat{q}_2 = \frac{\beta M_1 \hat{q}_0}{w},$$

where \( T = \text{scatter function}. \)

and

$$W_3(u) = [W_2(u) - \hat{q}_2]|T(u)|^2 + \hat{q}_2,$$

$$= \beta^2 \frac{M_2}{w} \frac{M_1}{w} |T(u)|^2 + \frac{\beta M_1 \hat{q}_0}{w},$$

where \( T(u) \) is the Fourier transform of the scatter point-spread function.

**II.B.4. Collection of charges in detector elements**

The detector generates a signal \( \tilde{d} \) proportional to the number of charge carriers that are collected in each detector element, where

$$\tilde{d} = k a_s a_{\tilde{d}} = k a_s \frac{\beta M_1 \hat{q}_0}{w},$$

(untl), and the presampling NPS (Ref. 10) is given by

$$W_d(u) = k^2 W_3(u) a_s^2 \text{sinc}^2(a_s u_t) a_s^2 \text{sinc}^2(a_s u_y)$$

$$= k^2 \beta^2 \hat{q}_0 \left[ \frac{M_2}{w^2} - \frac{M_1}{w^2} \right] |T(u)|^2 a_s^2 \text{sinc}^2(a_s u_t)$$

$$\times a_s^2 \text{sinc}^2(a_s u_y) + k^2 \frac{\beta M_1 \hat{q}_0}{w} a_s^2 \text{sinc}^2(a_s u_t)$$

$$\times a_s^2 \text{sinc}^2(a_s u_y),$$

in per square millimeter where \( u = (u_t, u_y). \)

**II.B.5. Detective quantum efficiency**

The DQE is therefore given by

$$\text{DQE}(u) = \frac{\tilde{d}^2 \text{MTF}^2(u)}{\hat{q}_0 W_d(u) + \sum_n \sum_m W_d \left( u \pm \left( \frac{x_n}{w}, \frac{y_m}{w} \right) \right)},$$

where the MTF is the product of \( T(u) \) and the sinc terms. The zero-frequency DQE value is then

$$\text{DQE}(0) = \frac{1}{M_2^2} \frac{1}{M_1^2} + \frac{w}{M_1^2} \left( 1 - \beta \right),$$

where \( \beta = \frac{M_2 - M_1^2}{M_1^2}. \) The above relationship expresses the zero-frequency DQE value as a function of moments of the AED obtained by Monte Carlo methods.

In Swank’s original work, he showed that

$$\text{DQE}(0) = \frac{1}{\alpha I_{AED}},$$

where \( \alpha = M_0 \) and where

$$I_{AED} = \frac{M_1^2}{M_0 M_2},$$

describes variations in the absorption process, sometimes just called the Swank factor. While this result has been used in a many subsequent investigations, our result shows that secondary quantum noise can be incorporated into the DQE calculation as

$$\text{DQE}(0) = \frac{1}{\alpha I_{AED}} + \frac{w}{M_1^2} \left( 1 - \beta \right),$$

which includes a second term in the denominator and accounts for secondary quantum noise. Equation (19) shows that low conversion gain or poor collection efficiency will degrade the DQE if the noise term exceeds a value of approximately 0.1, giving the requirement that

$$\beta > \frac{1}{1 + \frac{M_1}{10 w}}$$

to avoid a secondary quantum sink. This consideration will be most important at low x-ray energies.
III. METHODS

III.A. Photoconductor materials

The performance of seven candidate semiconductor materials for photoconductor-based mammography is evaluated using MC methods to determine the quantum efficiency \(\eta\) and Swank noise term \(\text{IAED}\). All material information was taken from Kabir et al.\(^47\) and is summarized in Table I. All materials, except Si, are assumed to be prepared as amorphous or polycrystalline forms and have reduced physical densities compared to crystalline structures.

III.B. Monte Carlo simulations

We carried out virtual pulse-height spectroscopy measurements using the latest version of Monte Carlo N-Particle transport code (MCNP version 5, the Radiation Safety Information Computational Center or RSICC, Oak Ridge, TN) to simulate the coupled photon–electron transport within a detector. X-ray photon energies were sampled from three representative mammographic spectra (Mo = Mo, Rh = Rh, and W = Al).\(^48\) Each spectrum was computer-generated by adjusting tube voltages to achieve the required half-value layers.\(^49\) Detailed spectral information is summarized in Table II. In all cases, a pencil-like photon beam was perpendicularly incident upon the center point of the top surface of the detector.

We modeled two different detector geometries, as illustrated in Fig. 3: (1) A cylindrical slab geometry having a radius of 200 mm and (2) a hexahedral geometry having an aperture of \(0.1 \times 0.1\) mm. In the former, we expect no lateral escape of characteristic or Compton scatter x-rays and instead expect escape only from the top and bottom surfaces. This geometry can be regarded as an “infinitely-sized” pixel element and we refer to it as the “\(a_\infty\)” model. The hexahedral geometry mimics typical pixel apertures ( \(\sim 100 \mu\)m) in mammography detectors and we refer to it as the “\(a_{100}\)” model. We mainly report on the results of the \(a_\infty\) model and the signal and noise characteristics on the \(a_{100}\) model are indirectly addressed by calculating the relative difference with respect to the \(a_\infty\) model

\[
\Delta \text{DQE} = \frac{\text{DQE}_{a_\infty} - \text{DQE}_{a_{100}}}{\text{DQE}_{a_{100}}} \times 100\%.
\]

A wide range of mass loading \((\rho \times L)\) values was considered for the Monte Carlo simulations. The maximum mass loading was set to \(\sim 230 \text{ mg cm}^{-2}\) (corresponding to \(L = 1 \text{ mm}\) for Si). Each simulation included \(2 \times 10^6\) incident photon histories with energy-absorbing events summed in 1-keV energy bins.

III.C. Calculation of the DQE from the AED

Evaluation of the DQE with Eqs. (18) and (19) requires the moments \(M_1\) and \(M_2\), evaluated as

\[
M_j = \int_0^\infty E^j A(E) dE,
\]

for the \(j\)th moment, where

\[
A(E) = \int_0^\infty s(E') A(E, E') dE'.
\]

We also calculate the mean energy absorbed per x-ray interaction

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Table I. Summary of material and atomic elemental information. Physical densities of photoconductor materials, and atomic number and shell binding energies of each compositional element are summarized.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g cm(^{-3}))</th>
<th>Atomic number (Z)</th>
<th>KL</th>
<th>L(_2)</th>
<th>L(_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>2.33</td>
<td>14</td>
<td>1.84</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>a-Se</td>
<td>4.3</td>
<td>34</td>
<td>12.66</td>
<td>1.65</td>
<td>1.48</td>
</tr>
<tr>
<td>CdZnTe</td>
<td>5.8</td>
<td>48 (Cd)</td>
<td>26.71</td>
<td>4.02</td>
<td>3.73</td>
</tr>
<tr>
<td></td>
<td>30 (Zn)</td>
<td></td>
<td>9.66</td>
<td>1.19</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>52 (Te)</td>
<td></td>
<td>31.81</td>
<td>4.94</td>
<td>4.61</td>
</tr>
<tr>
<td>HgI(_2)</td>
<td>6.3</td>
<td>80 (Hg)</td>
<td>83.10</td>
<td>14.84</td>
<td>14.21</td>
</tr>
<tr>
<td></td>
<td>53 (I)</td>
<td></td>
<td>33.17</td>
<td>5.19</td>
<td>4.85</td>
</tr>
<tr>
<td>PbI(_2)</td>
<td>4.0</td>
<td>82 (Pb)</td>
<td>88.00</td>
<td>15.86</td>
<td>15.20</td>
</tr>
<tr>
<td>PbO</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TlBr</td>
<td>7.5</td>
<td>81 (Tl)</td>
<td>85.53</td>
<td>15.35</td>
<td>14.70</td>
</tr>
<tr>
<td></td>
<td>35 (Br)</td>
<td></td>
<td>13.47</td>
<td>1.78</td>
<td>1.60</td>
</tr>
</tbody>
</table>

Table II. Mammographic beam qualities suggested by IEC and the obtained beam qualities from the computational spectrum simulator.

<table>
<thead>
<tr>
<th>Beam quality</th>
<th>IEC</th>
<th>Computer generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>kVp</td>
<td>HVL (mmAl)</td>
<td>Adjusted kVp</td>
</tr>
<tr>
<td>Mo/Mo</td>
<td>28</td>
<td>0.600</td>
</tr>
<tr>
<td>Rh/Rh</td>
<td>28</td>
<td>0.740</td>
</tr>
<tr>
<td>W/Al</td>
<td>28</td>
<td>0.830</td>
</tr>
</tbody>
</table>

---

Fig. 3. Illustration of the two geometries used for the Monte Carlo simulations. The cylindrical slab with radius 200 mm mimics an infinite pixel element and the hexahedron with aperture of \(0.1 \times 0.1\) mm mimics a typical pixel element employed in mammography detectors.
Values of the effective energy $w$ required to liberate each electron-hole pair for each material are obtained from Refs. 25 and 47 and listed in Table III.

### III.D. Validation

#### III.D.1. Comparison with theory

We compared the DQE obtained from the Monte Carlo simulations with those calculated from a simple analytic approach using the XCOM photon cross-sections library (The National Institute of Standards and Technology, MD). We assume that only photoelectric interactions result in...
energy absorption events and all the photon energy is deposited. In this case the AED is given by

\[ A(E) = s(E) x(E) = s(E) \left( 1 - e^{-\mu_{tot}(E)L} \right), \]

(25)

and \( j \)th moment of the AED is given by

\[ M_j = \int_{0}^{\infty} s(E) \left( 1 - e^{-\mu_{tot}(E)L} \right) \frac{\mu_{pe}(E)}{\mu_{tot}(E)} E^j dE, \]

(26)

where \( \mu_{tot}, \mu_{pe}, \) and \( L \) denote the total linear attenuation coefficient, photoelectric attenuation coefficient, and thickness of the x-ray convertor, respectively. Equations (25) and (26) account for variability in image signal due to the spectral width of the incident photons, but not the escape of scattered photons, and therefore are expected to overestimate the DQE determined from the Monte Carlo data.

III.D.2. Comparison with energy absorption tally

We verified the energy-moments method by comparing the total energy absorption tally for each simulation with the first moment of the pulse-height distribution [Eq. (22)]. The largest discrepancy was 0.69%, corresponding to a 0.7 mm-thick Si detector with a Mo/Mo spectrum, indicating the MC simulations are reliable in terms of their statistical precision.

IV. RESULTS

IV.A. Absorbed energy distribution

Distributions of deposited energy obtained from the Monte Carlo simulations are illustrated in Figs. 4–6 for Mo, Rh and W target materials, respectively, for material

![Graphs illustrating absorbed energy distribution](image_url)
thicknesses of 0.05 mm and 0.2 mm. The events in $A(E)$ for $E < 12$ keV are primarily due to partial energy deposition from escape of Compton-scattered and characteristic photons. Energy absorption events in energy bins greater than $\sim 12$ keV increase with increasing thickness while events in lower bins decrease due to scatter self-absorption. The AED curves corresponding to Mo and Rh targets show more structure than those from W targets due to the sharp spectral peaks resulting from characteristic emissions from Mo and Rh. In all cases, these structures are primarily the result of escape of characteristic emissions ($K$ or $L$ emissions, depending on atomic number and spectral energy).

As a generalization, we expect DQE values to be largest for AED curves that are distributed over the smallest energy range, indicating a complex relationship between detector material and spectral shape.

IV.B. Quantum absorption efficiency

Figure 7 illustrates the quantum efficiency $\langle\alpha\rangle$, for each photoconductor material and spectral combination as a function of mass loading calculated both theoretically and from MC data. Excellent agreement is obtained. The quantum efficiency of materials with higher atomic number $Z$ and density $\rho$ shows a larger dependence on convertor thickness than those with lower atomic number and density with the exception of $a$-Se which has lower $Z$ and $\rho$ than CdZnTe but demonstrates a larger dependence on convertor thickness because of its lower $K$-shell binding energy.

![Fig. 6. Computer-generated W/Al spectrum and AED curves of various photoconductors obtained from Monte Carlo simulations.](image-url)
IV.C. Energy deposition per interaction

The average deposited energy per interacting x-ray quantum for each photoconductor material is plotted as a function of mass loading in Fig. 8. Mean deposited energy increases with increasing average energy of the incident x-ray spectrum indicated by dashed lines in Fig. 8 and, as expected, never exceeds the average energy of the incident x-ray spectrum.

The dependence of deposited energy on the detector material is different from that of quantum absorption efficiency which is dependent upon both $Z$ and $\rho$. HgI$_2$, PbI$_2$, and CdZnTe demonstrate the largest increase in deposited energy with increasing mass loading and $\alpha$-Se and Si the smallest. The deposited energy of each material increases as the fraction of photoelectric absorption events per x-ray interaction increases. Partial energy deposition due to escape of characteristic and Compton-scattered photons reduces the mean deposited energy from the analytic model using Eq. (26).

IV.D. Swank noise term

Figure 9 illustrates the Swank factor $I_{AED}$ for the various photoconductor materials. In general, the dependence of the Swank factor on mass loading is similar to that of the mean deposited energy per interaction with the exception of Si. As illustrated in Figs. 4–6, additional peaks in lower energy bins produced by photon escape distort $A(E)$, and hence increase the variability in deposited photon energy and decrease the Swank noise term.

As shown in Fig. 9, $I_{AED}$ increases with mass loading and then plateaus. It is interesting to note that $I_{AED}$ of high-$Z$ materials decreases slightly with larger mass loading. This observation is more pronounced in HgI$_2$ and PbI$_2$ for the Mo spectrum, likely due to increased interaction probability of high-energy photons resulting in an increase in absorption events due to complete absorption of primary photons and partial reabsorption of secondary photons. This has the effect of increasing the variability in deposited energy and decreasing the Swank noise term.

IV.E. Zero-frequency DQE

The minimum collection efficiency required to avoid a zero-frequency secondary quantum sink$^{12}$ as determined by Eq. (20) are summarized in Table III. In summary, an efficiency of 0.02 is required for $\alpha$-Se and less than 0.01 for all other materials. At high spatial frequencies, this value is likely 10$^{-3}$ this value, or 0.2 for $\alpha$-Se and 0.1 for all other materials.
For conditions that avoid a secondary quantum sink, the zero-frequency DQE value determined using Eq. (19) is illustrated in Fig. 10. The best DQE performances are achieved with PbO and TlBr. For mass loading greater than 100 mg cm\(^{-2}\), a-Se, HgI\(_2\), and PbI\(_2\) provide similar DQE values to PbO and TlBr.

IV.F. Effect of pixel-aperture size on the signal and noise properties

While the zero-frequency DQE is slightly degraded (by 0\%–4\%) in the \(a_{100}\) model, as illustrated in Fig. 11, there is in general very little dependence on the aperture size. This suggests that lateral escape of characteristic and Compton-scattered photons does not severely reduce the zero-frequency DQE of the photoconductors investigated in this study for mammography. However, it should be noted that this calculation does not take into account the noise correlations introduced by reabsorption of characteristic and Compton scatter x-rays in neighboring pixels.

V. DISCUSSION

The MC-based model described here is the first that makes a direct link to the cascaded-systems approach including the effect of secondary quantum noise, and showed that approximately 20\% of charges liberated in a-Se and 10\% of charges liberated in other materials must be collected to avoid a secondary quantum sink. Under these conditions, the quantum efficiency, average energy absorption, Swank noise term, and DQE were derived from AED curves obtained from Monte Carlo simulations. We considered a geometry having a large enough aperture to neglect lateral escape of Compton-scattered and characteristic photons and a geometry having an aperture size of 0.1 \(\times\) 0.1 mm. Differences in the signal and noise properties between the two geometries were less than a few percent. These results are consistent with the works of Sakellaris et al.\(^{33,34}\) The large-area DQE is given by the product of the quantum efficiency and Swank noise terms, and since the range of \(IAED\) values in this study was relatively small, the effect on DQE was modest.

A common selection and design criteria for photoconductors for mammographic applications are to find high-Z, high-\(q\) materials to maximize the quantum efficiency. Although lower-Z, lower-\(\rho\) materials, such as a-Se, are easily made to have sufficient thickness to attain high quantum absorption efficiency and DQE, this can result in loss of spatial resolution due to oblique angle effects. Another design strategy might be to use high-\(\rho\) materials that have a \(K\)-edge energy within the range of photon energies of the incident spectrum. This would have the effect of increasing

Fig. 9. Swank noise factors (\(IAED\)) of various photoconductor materials obtained from Monte Carlo simulations as a function of mass loading for different mammography spectra.

Fig. 10. Calculated zero-frequency DQE of various photoconductor materials as a function of mass loading for different mammography spectra.
the quantum efficiency while at the same time reducing the migration of characteristic photons. Of the materials considered in this study, PbO had the highest zero-frequency DQE for each of the x-ray spectra considered.

If a detector can be made to operate in an energy-discriminating photon-counting mode, rather than conventional energy-integrating, the selection of materials having a higher Swank factor is of great importance because the Swank factor is directly related to variability in the measurement of photon energy. For the design of mammography detectors capable of energy discrimination, therefore, it would be preferred to select materials having a large photoelectric-to-Compton interaction ratio.

The photon-counting approach with Si microstrip linear-array detectors has recently been implemented in a multi-slit-scanned digital mammography system. This system implements a 3.6-mm thick “edge-on geometry” to increase the quantum absorption efficiency. While we did not consider thicknesses greater than 1 mm for Si, we expect that increasing the thickness to 3.6 mm could result in a 15%–20% increase in the zero-frequency DQE and Si could therefore have similar DQE values to PbO. The DQE reported here is greater than their reported value of approximately 0.7, possibly due to other factors affecting the Swank factor but not included in our MC calculations such as the charge-sharing effect that can significantly reduce the Swank factor. 

Mainprize et al. described a theoretical DQE model and added a noise term assuming independent shot noise. They investigated the effect of incomplete charge collection and showed the DQE could be reduced by as much as 50% if the trapping density is high, analogous to the secondary quantum sink reported here. Similar results have been reported by others. Charge collection properties are determined by the material quality, polarity and the bias voltage, known as the mean drift length ($\mu_j F$ where $\mu_j$ and $F$ denote mobility and lifetime for the charge carrier of $j$, respectively, and $F$ the electric field intensity). and CdZnTe and HgI$_2$ show the best characteristics. Although an increase in bias voltage can enhance the charge collection properties, it can also elevate leakage current which plays a role as shot noise. Considering these factors, HgI$_2$ could be a good choice of photoconductor for mammography.

Alternatively, consideration of a thinner photoconductor with a higher quantum absorption efficiency might be a solution to mitigate incomplete charge collection under operation in lower bias voltage. Again, PbO would be a good candidate.

VI. CONCLUSIONS

The signal-to-noise characteristics of various photoconductor materials for standard mammographic spectra were determined using Monte Carlo methods incorporated into a cascaded-systems model. The model provides a condition showing that 10%–20% of all liberated quanta must be collected to avoid a secondary quantum sink. The quantum efficiency, average deposited energy per interacting x-ray, Swank noise factor, and zero-frequency DQE are tabulated by means of graphs that may help in the design and selection of materials for photoconductor-based mammography detectors. Neglecting the electrical characteristics of photoconductor materials, and taking into consideration only the physics of x-ray interactions, it is concluded that, of materials considered in this study, PbO shows the strongest zero-frequency DQE performance. When electrical characteristics, such as secondary quantum gain, charge collection and dark current are considered, HgI$_2$ may also be an excellent photoconductor for mammography.

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APPENDIX: SIGNAL AND NOISE TRANSFER THROUGH A PHOTON-INTERACTION PROCESS

In this section, transfer functions describing propagation of the mean $\bar{q}$ and NPS $W(u)$ through a process that combines both selection and gain stages are developed theoretically. Both the probability of interaction and mean conversion gain
are functions of a random variable describing the incident photon energy. The mean and NPS can be described using the input-labeled transfer relationships developed by Rabbani and Van Metter:

\[ q_{\text{out}} = \langle m \rangle, q_{\text{in}} = \langle g \rangle, q_{\text{in}}, \]  

(A1)

where \( m = g \) is the energy-dependent mean gain of the combined process and

\[ \langle g \rangle = \int_0^\infty s(E') \gamma(E') g(E') dE', \]  

(A2)

denotes the average of \( \gamma(E') g(E') \) weighted by the distribution of incident x-ray photon energies \( s(E') \).

The NPS is given by

\[ W_{\text{out}}(u) = \langle m \rangle^2 |W_{\text{in}}(u) - \langle q \rangle| + \langle \sigma_m^2 + \langle m \rangle^2 \rangle, \]  

(A3)

Using the product rule to show \( \sigma_m^2 = x^2 \sigma_q^2 + y^2 \sigma_q^2 + z^2 \sigma_q^2 \) results in Eq. (2).

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