Modeling of pulse signals in photon-counting detectors

Chang Hwy Lim\textsuperscript{a}, Okla Joe\textsuperscript{a}, Ian Cunningham\textsuperscript{b}, Ho Kyung Kim\textsuperscript{a}\textsuperscript{*}

\textsuperscript{a} School of Mechanical Engineering, Pusan National University, Busan 609-735, South Korea
\textsuperscript{b} Imaging Research Labs, Robarts Research Institute, 100 Perth Drive, London, Ontario N6A 5K8, Canada

ABSTRACT

We are developing a theoretical model to describe signal pulses from a detector-amplifier system operated in photon-counting mode. In the model, we include incomplete signal generation due to the charge trapping within a semiconductor detector as well as to the ballistic deficit caused by insufficient charge integration time. This model can be utilized for the characterization of detector material properties such as the mobility and the lifetime, as well as the optimization of operation conditions such as the applied bias voltages and the charge integration time. The model was experimentally verified with the measurement of charge collection efficiency of a planar cadmium zinc telluride detector with respect to the applied bias voltage and the charge integration time. We expect that the developed model will be helpful for the design of photon-counting detectors.

Keywords: Cadmium zinc telluride, charge collection efficiency, charge integration, photon counting, pulse, shaping amplifier, wide bandgap detectors

1. INTRODUCTION

In diagnostic x-ray radiology, numerous digital detector technologies have been designed and some of them are now successfully used.\textsuperscript{1} Efforts are still being devoted for further improvements in detector performance and reduced patient dose. In the line with this, several new detector technologies have recently been investigated.\textsuperscript{2} Among them, energy-specific detectors, in which the energy of each interacting x-ray photon can be determined, are greatly paid attention.\textsuperscript{3} The conventional digital radiography detectors operate in integration mode, \textit{i.e.}, the detectors integrate the incoming signal over the time of the x-ray exposure to acquire an image. Therefore, the resultant image signal is proportional to the deposited energy rather than the number of individual x-ray photons. In this case, we cannot avoid the noise factor due to the distribution of energy deposition in the x-ray converters first identified by Swank,\textsuperscript{4} even when the incident x-ray photon is monochromatic. The energy-dependent response of a detector and its related noise can be reduced by operating the detector in the counting mode. Therefore, the photon-counting method has a great potential of reduction in patient dose.\textsuperscript{5}

Actually, this technology is not new because the detector system in nuclear medicine mostly employs this concept. However, the design of energy-specific detectors for x-rays is challenging since we have to deal with an x-ray photon individually from high flux of incident x-rays.\textsuperscript{5,6,7} Each pixel element of the two-dimensional array for imaging may require a highly elaborate detector technology to incorporate a charge-sensitive preamplifier (CSPA), a shaping amplifier, discriminators, shift registers, digital-to-analog converters, and digital logic.\textsuperscript{5,7} There is a review article available covering the recent developments in the area of semiconductor photon-counting detectors.\textsuperscript{3}

In this study, we have developed a simple analytical model to describe signal pulses from a typical detector-amplifier system employed in common photon-counting systems. In the model, we have included incomplete signal generation due to the charge trapping within a semiconductor detector as well as to the ballistic deficit caused by insufficient charge integration time. The developed model has experimentally been verified with the measurement of charge collection efficiency (CCE) of a planar cadmium zinc telluride (CdZnTe) detector. We expect that the developed model will be helpful for the design of a photon-counting detector.

\textsuperscript{*} hokyung@pnu.edu; phone +82 51 510 3511; fax +82 51 518 4613
2. MATERIALS AND METHODS

2.1 Simple detector-amplifier system for photon-counting measurements

Silicon or germanium has the longest history as a semiconductor detector for radiation detection. Recently, compound semiconductors, such as cadmium telluride (CdTe), CdZnTe, mercuric iodide (HgI₂), and lead iodide (PbI₂), are greatly paid attention because of a high atomic number and a wide bandgap, which provide a large value of quantum efficiency and the low leakage current, respectively. However, the compound semiconductor suffers from incomplete charge collection with a large fluctuation due to trapping-detrapping of charge carriers, which gives rise to poor energy resolution in spectroscopy and to an image lag in the digital x-ray imaging. Polycrystalline structure for large-area application worsens charge collection properties because of a poorer charge transport properties. In this study, a single detector is assumed to have simple parallel-plate geometry.

The signal-processing system applied to count a single photon with an energy usually consists of a preamplifier, a shaping amplifier and a single-channel analyzer. To specify photon energies from polyenergetic x-rays, a multi-channel analyzer (MCA) is required. Fig. 1 describes a simple detector-amplifier system, considered in this study, and which is a typical layout for gamma-ray spectroscopy.

The CSPA is used to amplify and produce a voltage pulse directly proportional to the signal charge generated in a detector. The shaping amplifier may further amplify the output voltage pulse from the CSPA, but the main role is to shape the long-tailed CSPA pulses, which are due to slow decay of the charge in the feedback capacitor through a high resistance in parallel with the capacitor, into short pulses. Without shaping into short pulses, the well-known "pulse pile-up" can occur and this superimposed pulse no longer delivers the measured energy in the detector into the next signal-processing element exactly. The pulse pile-up becomes serious when detection rate increases.

Another important role of the shaping amplifier is to improve signal-to-noise ratio by eliminating electronic noise of low and high frequencies compared to the characteristic frequency of the signal pulse. Although there are many candidates for a shaping amplifier, we consider a successive high-pass and low-pass filter networks in a form of CR-(RC)^n, where n denotes the number of RC filters.

The MCA is a combination of an analog-to-digital converter and an addressable memory and it displays the distribution of pulse counts in a digitized scale of the pulse heights or channels.

2.2 Signal formation in a detector-amplifier system

The modeling of signal generation in semiconductor detectors is based on the previous study. The charge collection in a planar semiconductor detector is described by the well known Hecht equation, in which the incomplete charge collection...
is handled by the simple deep-trapping approximation and the concept of mean lifetime.\(^9\) We apply the Hecht equation to the case of a distributed charge generation through a detector by the incident photon radiation in a direction perpendicular to the electrodes. Total induced charge at the output node of the detector for a given detector thickness \(L\) and a charge collection (or integration) time \(\tau\), is given by integrating the induced current due to \(n_0\) charges generated at an initial position \(x_0\):

\[
Q_j = \int_{L}^{0} dx_0 \int_{0}^{\tau} dt I_j(x_0,t),
\]

\[
I_j(x_0,t) = \frac{q \mu_j V_A}{L^2} n_0(x_0) e^{-t/\tau_j} = q n_0(x_0) \frac{\lambda_j}{L} e^{-t/\tau_j},
\]

where, the subscript \(j\) designates the corresponding charge carrier, viz., \(e\) and \(h\) for the electron and hole, respectively. \(\mu\) and \(\tau\) describe the mobility and the lifetime of the charge carrier, respectively. \(q\) is the electronic charge and \(V_A\) is the applied bias voltage. \(\lambda\) describes the mean drift length or \(\mu \tau F\), where \(F\) is the electric field intensity. It is noted that the number of charge carriers generated at a position \(x_0\) has a unit of \(\text{cm}^{-1}\). We can then estimate the CCE as

\[
\eta = \frac{Q_C}{Q_0} = \frac{Q_e + Q_h}{Q_0} = \eta_e + \eta_h,
\]

where \(Q_C\) and \(Q_0\) are the collected and generated charges, respectively.

The transit time larger than the charge integration time causes a "ballistic deficit".\(^8\) In such cases, as described in Fig. 2, some of the charges are regarded as fully collected charges whereas the others are not. However, it is important to note that the charges other than the fully collected still contribute to the induced charge (or signal) by the drift motion during the charge integration time. This fractional signal contribution should be considered by identifying the location of the non-fully collected charges. The detailed formalism describing the calculation of CCE is found in Ref. 9.

We consider the output pulse decay from the CSPA through the feedback resistance \(R_f\) with a decay time constant, \(\tau_{pre} = R_f \times C_f\). If we simplify the CSPA to a simple parallel RC network,\(^10\) the corresponding output voltage pulse becomes simply

\[
V_{pre,j}(t) = \frac{1}{C_f} e^{-t/\tau_{pre}} \int_{0}^{t} dt' I_j(t') e^{t'/\tau_{pre}}.
\]

If the decay time constant of the CSPA is large enough not to decay, then the output pulse is directly proportional to the collected charges.

Fig. 2. Schematics depicting charge generation in a semiconductor detector with a parallel plate geometry. Transport directions of the generated charge carriers are indicated for two different configurations: (a) negatively biased radiation-receiving electrode and (b) positively biased radiation-receiving electrode. Boundary positions of the charge carriers that never reach to the corresponding electrode for a given charge-collection time are indicated.
The output pulse signal of the CR-(RC)\textsuperscript{n} shaping amplifier can be calculated by a signal differentiation and \(n\) integration of the output pulse from the CSPA such that

\[
V_{\text{amp},j}(t) = \int_0^t \int_0^t \cdots \int_0^t \frac{dV_{\text{pre},j}(t)}{dt} dt = \int_0^\infty \sum_{n=1}^\infty \frac{\tau_{\text{amp}}^n}{\tau_{\text{pre}}^n} \frac{1}{(k-1)!} t^{k-1} e^{-t/\tau_{\text{pre}}} \frac{t^n}{\tau_{\text{amp}}^n} e^{-t/\tau_{\text{RC}}} dt.
\]

If the each time constant constituting the successive CR-(RC)\textsuperscript{n} networks has the same value as \(\tau_{\text{RC}}\), the output voltage pulse from the CR-(RC)\textsuperscript{n} network might be calculated by using the Laplace transform:

\[
V_{\text{amp},j}(t) = \frac{\tau_{\text{amp}}}{\tau_{\text{pre}}^n \tau_{\text{RC}}} e^{-t/\tau_{\text{RC}}} \sum_{k=1}^n \frac{\tau_{\text{amp}}^k}{\tau_{\text{pre}}^n} \frac{1}{(k-1)!} t^{k-1} e^{-t/\tau_{\text{pre}}} \frac{t^n}{\tau_{\text{amp}}^n} e^{-t/\tau_{\text{RC}}},
\]

where

\[
\tau_{\text{amp}} = \frac{\tau_{\text{pre}} \tau_{\text{RC}}}{\tau_{\text{pre}} - \tau_{\text{RC}}}.
\]

2.3 Experimental

To validate the developed model, we have estimated the CCE in a CdZnTe detector for \(^{57}\text{Co}\) radioisotope source mainly emitting gamma photon of 122 keV. The planar CdZnTe detector (eV-Microelectronics, Inc., USA) has a dimension of \(10 \times 10 \times 5\) mm\textsuperscript{3}. To observe the polarity-dependency, the photon-receiving electrode was negatively or positively biased from 500 – 1000 V. The incident photon was shaped into the pencil beam with a diameter of 2.5 mm by using a 2-mm-thick lead collimator.

To acquire pulse-height distributions from the detector, we have constructed the measurement system with a CSPA (eV-550, eV-Microelectronics, Inc., USA), a linear shaping amplifier (572A, Ortec, USA), and an MCA (926, Ortec, USA), as described in Fig. 1. The photon-induced charge signal from the detector was capacitively fed into the CSPA. The signal pulse from the CSPA was then shaped by the shaping amplifier with a wide range of shaping (or collection) time (0.5 – 10 \(\mu\)s) and finally the shaped signal was processed by the MCA. Linearity verification as well as the gain calibration (from the channel number to electrons or simply \(e^+\)) of the cascaded signal processing chain was performed by introducing known charge pulses from a test pulser (480, Ortec, USA) into the test input of the CSPA.

3. RESULTS AND DISCUSSION

Fig. 3 shows pulse-height spectra obtained from a \(^{57}\text{Co}\) source using a CdZnTe detector having a thickness of 5 mm for various applied bias voltages and shaping times. The increase of pulse height or collected charges is clearly observed as
the bias voltage or the shaping time increases. Therefore, it is important to estimate signal generation or charge collection in the detector-amplifier system for the better design in terms of geometry and operation condition.

With the material parameters of \( \mu_e \tau_e = 3 \times 10^{-3} \text{ cm}^2/\text{V} \) and \( \mu_h \tau_h = 8 \times 10^{-5} \text{ cm}^2/\text{V} \), which were provided by the detector manufacturer, the estimated charge collection efficiencies with respect to the collection time and the applied bias are shown in Fig. 4. The developed analytical model gives reasonable agreement with the measurements. If we iterate the calculations considering \( \mu_e \tau_e \) values as variables, then we would have more well-agreed results. This implies that the developed model can be used for characterizing detector material properties, although time-of-flight measurements with a radiation source having a very high stopping power, such as alpha particles, are typically used. The developed model

![Fig. 4. Charge collection efficiencies of the CdZnTe detector for 122-keV photons. Circles are the experimental measurements and solid lines are the calculation results. Charge collection efficiencies as a function of the collection time and the applied bias voltage are, respectively, shown in the upper and lower panels. In these calculations the material parameters, such as \( \mu \tau \) values, were provided by the detector manufacturer, and not optimized to match with the measurements in this study.](image1)

![Fig. 5. Comparisons between the measured and calculated pulse-height spectra. The calculation fits the tail of photopeak with reasonable agreement.](image2)
can be easily extended to polyenergetic source, e.g. x-ray spectrum from a tube, if we know the spectral information exactly.

Assuming the exponential event distribution, i.e. the Lambert-Beers law, within a detector for the incident photons, we can calculate the pulse-height distribution with the charge collection model as demonstrated in Fig. 5. In these calculations, the spectral broadening due to the electronic noise of the system was considered by convolving the calculated results with the measured noise. Instead of the given $W$-value (or the average energy needed to create a single electron-hole pair) of 4.64 eV by the manufacturer, we used slightly higher value (~15%) to match the peak channel. If we use more precise values describing material parameters, which can be extracted by comparing the measured and calculated charge collection efficiencies iteratively, we would have better matched spectra.

Long tail extending to lower channels is due to the smaller value of $\mu_h \tau_h$ because the measurement system in this study is sensitive to both the positive and negative signal polarities. If we can only accept a faster (or larger) signal pulse, then we have very symmetric photopeak without tail into lower pulse-height channel. The method for single-polarity charge sensing with semiconductor detectors was already introduced, which uses coplanar electrodes to emulate the function of Frisch grids commonly employed in gas and liquid ionization detectors.\textsuperscript{11}

With the provided material parameters, signals in various physical quantities from the photon-counting system, which consists of the CdZnTe detector having a thickness of 5 mm, the CSPA with a time constant, $\tau_{\text{pre}}$, of 250 $\mu$s, and a CR-$(RC)^4$ shaping amplifier with a time constant, $\tau_{\text{RC}}$, of 0.125 $\mu$s (or the collection time $\tau_c = n \times \tau_{\text{RC}} = 0.5$ $\mu$s), were

![Fig. 6. Calculated signals in various physical quantities from a photon-counting system. The calculated signals are the average values considering the Lambert-Beers law for photon interactions within a detector. (a) Average collection efficiency of charge carriers generated in the detector during a fixed collection time. (b) Average current induced by the movements of charge carriers. (c) Average output voltage signal from the charge-sensitive preamplifier with $\tau_{\text{pre}} = 250$ $\mu$s. (d) Average output voltage signal from the shaping amplifier. For comparisons, output voltage signals due to electrons from the charge-sensitive preamplifier and the shaping amplifier when the time constant of the charge-sensitive preamplifier is reduced by a factor of 100 are also plotted in (c) and (d). Solid, dotted, and dashed lines designate each contribution by electrons, holes, and both charge carriers, respectively. In these calculations, the detector-amplifier system was operated with the applied bias of 700 V and the collection time of 0.5 $\mu$s.](image-url)
calculated. The calculated signals are the average values considering the Lambert-Beers law for photon interactions within a detector. The applied bias was assumed to 700 V. The CCE shown in Fig. 4 is replotted in Fig. 6(a). As shown in Fig. 6(b) the current induced by the electron motion is confined within the given collection time of 0.5 μs, which means that the transit time of electrons generated at any position in the detector is less than the given collection time, hence the CCE of electrons saturates before the collection time (full collection) as shown in Fig. 6(a). It should be noted that, however, the CCE of ~0.5 does not mean the complete collection of electrons generated by photon interactions because the model incorporates deep-charge trapping. The complete collection efficiency would be higher than 0.5. On the contrary, the current induced by holes exists after the collection time because of slow mobility, as shown in Fig. 6(b). Therefore, in order to fully collect holes generated in the detector, much longer collection time is needed as shown in Fig. 6(a).

The properties of charge collection are directly reflected to the voltage pulse signals from the CSPA when its time constant is long enough to integrate the generated charges. However, if the time constant of CSPA is shorter, the signal loss may occur, and which is well known as the *ballistic deficit*. The signal loss of electrons from the CSPA having a short time constant (e.g. $\tau_{pre} = 2.5 \, \mu$s) is demonstrated in Fig. 6(c).

The voltage pulse signals from the CR-(RC)$^n$ shaping amplifier (shortly, shaper) are calculated and plotted in Fig. 6(d). The peaking time of the electron pulse corresponds to the collection time because the electrons were fully collected within the given collection time. However, the peaking time of the pulse due to the total charge carriers slightly exceeds the collection time because of hole collection. The signal loss due to the shorter time constant of the CSPA is expressed as the reduced pulse height and undershoot of the voltage pulse signal from the shaper as shown in Fig. 6(d). Therefore, the proper design of CSPA is very important to avoid an additional signal loss except the physical limitation in a selected detector due to the detector material parameters.

Fig. 7 shows the calculated output signal pulse shape of CR-(RC)$^n$ shaping amplifier with $n = 1$–10 at a shaping time of 0.5 μs. If the integration stage (or low-pass filter) is increased, the shape of pulse becomes to a Gaussian distribution, which has the best signal-to-noise ratio. However, the realization of shaping amplifier with many integration stages is complex, especially when we consider pixel-level realization for digital x-ray imaging. Approximate Gaussian shape, for example $n = 4$, is typically incorporated into commercially available shaping amplifier.

4. CONCLUSIONS

Simple analytical model describing signals in the cascaded signal-processing chains constituting a simple photon-counting measurement system has been developed and validated with experimentally measured pulse-height distributions and estimated CCEs. Although the developed model assumes and thus incorporates very simplified
electronic circuit models for the CSPA and the shaping amplifier, some important observations have been made: The charge collection characteristic in a detector is largely dependent on the transport properties of charge carriers and thus the CSPA should be designed to sensitive to a dominant polarity; In order to reduce the additional loss of signal, the decay time constant of the CSPA should be large enough; The slow component in charge carriers can alter the peaking time at the shaping amplifier; And four RC networks in the simple CR-(RC)^n shaping amplifier provide a reasonable signal-to-noise ratio. We expect that the developed model will be helpful for the design of photon-counting detectors.

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REFERENCES