Pulse Electronics

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• Assuming that the detector is operated in **pulse mode** with applications that involve either:
  – simple **counting** of radiation-induced events, resulting in the **count rate**;
  – or recording the distribution of energy deposited in the detector (**spectroscopy**), resulting in the **energy spectrum**

• \[ E_0 \Rightarrow \tilde{E}_{dep} \Rightarrow \tilde{Q}(\tilde{E}_{dep}) \Rightarrow \tilde{I}(\tilde{t}; \tilde{E}_{dep}) \Rightarrow \tilde{V}(\tilde{t}; \tilde{E}_{dep}) \]
  – Note that the overhead tilde indicates random variable
  – Usually too small \( Q \sim fC \) to be sensed directly
The preamplifier usually has a charge sensitive configuration, integrating the transient $i_{sig}(t)$ pulse to produce a voltage step $\Delta V$ proportional to $Q$.

The shaping amplifier converts the preamplifier output signal into an output voltage pulse with pulse height $V_{\text{peak}}$.

Integral discriminator selects all pulses with $V_{\text{peak}}$ above a certain threshold; Differential discriminator (or single channel analyzer) selects pulses with $V_{\text{peak}}$ between upper and lower thresholds.

Multichannel analyzer (MCA) measures the pulse height for each of a series of selected events, assigning the height to one of many pulse height ranges or channels, & incrementing a counter (or memory register) for the appropriate channel.
• Drift motion of e-h pairs generates $i_{\text{sig}}(t)$
• CSPA integrates $i_{\text{sig}}(t)$ onto the $C_F$ ($\sim\text{pF}$), producing $\Delta V$ ($\sim\text{mV}$) = $Q/C_F$ & restores $i_{\text{sig}}(t)$ to ground by using $R_F$ ($\sim10^2 \text{ k}\Omega$) with a tail pulse (rapid rise & very slow return, $\tau = R_F C_F = \sim10^2 \mu\text{s}$)
• **High-pass filter** or **differentiator** of the shaping amplifier passes the rising edge, but then the signal returns rapidly to baseline
• Amplification ($G$) of differentiated voltage pulses
• **Low-pass filter** or **integrator** improves the SNR
• **Rise** and **fall** times of $V_{SA}(t)$ have $\sim\tau_{\text{int}}$ & $\sim\tau_{\text{diff}}$, respectively, and $\tau_{\text{int}} \approx \tau_{\text{diff}}$ yields $\tau$
Major electronic considerations

• Pulse pile-up vs. count rates

• Electronic noise => SNR of pulses => energy resolution

• Baseline shift => accuracy of pulse amplitude

• Drift in pulse amplitude => energy resolution
  – Spectrum stabilization
Preamplifier

- Not needed for G-M tubes and scintillation counters due to large $Q$

- Provided as an *interface* btwn the detector & the pulse-processing electronics
- Should be located as close as possible to the detector to minimize the capacitive loading (noise source!) on the preamp
- Terminating the capacitance quickly; maximizing the SNR
- Driving its signal into the large capacitance represented by the long interconnecting cable (equivalently, the preamp should have a low output impedance)
- Having a *short rise time* consistent w/ the charge collection time in the detector
- Having a *long decay time* to consider various detectors w/ widely differing charge collection times before decay of the pulse sets in
Voltage-sensitive preamp

- Historically more conventional
- \( V_{\text{max}} = \frac{Q}{C_{\text{in}}} \) when \( \tau_{\text{in}} > \tau_{\text{col}} \)
  - No longer holding the const. proportionality if \( C_{\text{in}} \) changes
  - The cap. of semiconductor detectors changes w/ operating parameters

\[ V_{\text{out}} \approx -\frac{R_2}{R_1} V_{\text{in}} \]

Assume \( A >> R_2/R_1 \)
Charge-sensitive preamp

- Not affected by changes in $C_{in}$
- $V_{out} \cong \frac{Q}{C_F}$ as long as the duration of $i_{sig}(t) < \tau_F = R_F C_F$
- (Ideally) the rise time is determined only by $\tau_{col}$ in the detector; and is indep. of $C_{in}$ of the detector & preamp input

![Diagram of charge-sensitive preamp circuit](image)

\[
\begin{align*}
\text{Assume } A & \gg (C_i + C_F)/C_f \\
V_{out} & = -A \cdot V_{in} \\
V_{out} & = -A \frac{Q}{C_i + (A + 1)C_f} \\
V_{out} & \cong -\frac{Q}{C_f}
\end{align*}
\]
Shaping amplifier

- Should return the output pulse rapidly to the *baseline* to prevent pulses from overlapping and a resulting distortion of the measurement
- Filtering the high- and low-frequency noise to improve the SNR
- Characterized by the shaping time constant, \( \tau \)
  - Short \( \tau \) can minimize pulse pile-up but pass more broadband electronic noise

\[ \tau = R_F C_F = 50\sim 100\ \mu s \]

From a preamp

After shaping the pulse train
**CR-RC shaping amp**

- CR differentiator (or HPF) + RC integrator (or LPF)
- CR network

\[
E_{in} - \frac{Q}{C} - E_{out} = 0 \quad \text{(KVL)}
\]

Differentiating:
\[
\frac{dE_{in}}{dt} = \frac{1}{C} \frac{dQ}{dt} + \frac{dE_{out}}{dt} = \frac{1}{C} i + \frac{dE_{out}}{dt}
\]

Noting that \( E_{out} = iR \) and \( \tau = RC \);
\[
E_{out} + \tau \frac{dE_{out}}{dt} = \tau \frac{dE_{in}}{dt}
\]

\[\text{Solving the 1st-order linear DE:}\]
\[y' + p(x)y = r(x)\]
\[y(x) = e^{-h} \left[ \int e^{h} r(x) dx + c \right] \]
where \( h = \int p(x) dx \)

**General solution:**
\[E_{out}(t) = E_{out}(0) e^{-t/\tau} + e^{-t/\tau} \int_{0}^{t} e^{-t'/\tau} \frac{dE_{in}(t')}{dt} dt'\]

**Intuitively**

- If \( \tau < \) the duration of \( i \) pulse;
  \[E_{out} \approx \tau \frac{dE_{in}}{dt} \quad \text{(differentiator)}\]

- If \( \tau > \) the duration of \( i \) pulse;
  \[\tau \frac{dE_{out}}{dt} \approx \tau \frac{dE_{in}}{dt}\]
For a sinusoidal $E_{in} = E_i \sin(2\pi ft)$;  

$$ \frac{E_{out}}{E_{in}} = |A| \sin(2\pi ft + \theta) $$

where  

$$ |A| = \frac{1}{\left[1 + (\frac{f_1}{f})^2\right]^{1/2}} $$

$$ \theta = \tan^{-1}\left(\frac{f_1}{f}\right) $$

$$ f_1 = \frac{1}{2\pi \tau} $$

For a step voltage $E_{in} \begin{cases} E & (t \geq 0) \\ 0 & (t < 0) \end{cases}$;  

$$ E_{out} = E e^{-t/\tau} $$

Laplace transformation:  

$$ \hat{E}_{out} = A \hat{E}_{in} = \frac{R}{R + 1/sC} \frac{E}{s} = \frac{\tau E}{1 + s\tau} $$

$$ E_{out} = E e^{-t/\tau} $$
• RC network

\[ E_{in} = iR + E_{out} \]  
where KVL

\[ i = \frac{dQ}{dt} = C \frac{dV_c}{dt} = C \frac{dE_{out}}{dt} \]

Then, we have

\[ E_{in} = \tau \frac{dE_{out}}{dt} + E_{out} \]

Rearranging;

\[ \frac{dE_{out}}{dt} + \frac{1}{\tau} E_{out} = \frac{1}{\tau} E_{in} \]

General solution;

\[ E_{out}(t) = E_{out}(0)e^{-t/\tau} + \frac{1}{\tau} e^{-t/\tau} \int_0^t E_{in}(t')e^{t'/\tau} dt' \]

If \( \tau > \) the duration of \( i \) pulse;

\[ \frac{dE_{out}}{dt} \approx \frac{1}{\tau} E_{in} \text{ or } E_{out} \approx \frac{1}{\tau} \int E_{in} dt \] (integrator)

If \( \tau < \) the duration of \( i \) pulse;

\[ \frac{1}{\tau} E_{out} \approx \frac{1}{\tau} E_{in} \]
For a sinusoidal $E_{in} = E_i \sin(2\pi ft)$;

$$\frac{E_{out}}{E_{in}} = |A| \sin(2\pi ft + \theta)$$

where

$$|A| = \frac{1}{\left[1+(\frac{f}{f_2})^2\right]^{1/2}}$$

$$\theta = -\tan^{-1}\left(\frac{f}{f_2}\right)$$

$$f_2 = \frac{1}{2\pi \tau}$$

For a step voltage $E_{in} \begin{cases} E & (t \geq 0) \\ 0 & (t < 0) \end{cases}$;

$$E_{out} = E \left(1 - e^{-t/\tau}\right)$$

Laplace transformation:

$$\hat{E}_{out} = A\hat{E}_{in} = \frac{1/sC}{R+1/sC} \frac{E}{s} = \frac{1}{1+s\tau} \frac{E}{s} = \frac{E}{s(1+s\tau)} = E \left(\frac{1}{s} - \frac{\tau}{1+s\tau}\right)$$

$$E_{out} = E \left(1 - e^{-t/\tau}\right)$$
CR-RC shaping

- CR + ideal unity-gain amp (w/ $Z_{in} = \infty$ & $Z_{out} = 0$) + RC
- For a step voltage: $E_{out} = E \frac{\tau_{CR}}{\tau_{CR} - \tau_{RC}} (e^{-t/\tau_{CR}} - e^{-t/\tau_{RC}})$
  - $E_{out} = E \frac{t}{\tau} e^{-t/\tau}$ if $\tau_{CR} = \tau_{RC}$
- Trade-off in determining $\tau$
  - To reduce the pile-up effect, $\tau \downarrow$
  - To avoid the ballistic deficit, $\tau \uparrow (> \tau_{col})$
CR-(RC)^n shaping

- Gaussian shaping
  - \( E_{out} = \frac{E}{n!} \left( \frac{t}{\tau} \right)^n e^{-t/\tau} \)
  - \( n = 4 \) is sufficient
  - The peaking time, \( t_{peak} = n\tau \)
    - The time required for the shaped pulse to reach its max. amplitude

![Graph showing S/N ratios for different shaping methods.](image)
Pole-zero cancellation

- If the decay of the preamp is not infinite (i.e. not a step-like pulse), there will be a slight zero crossover or undershoot of the pulse from the shaping amp
  - The pulse recovers back to zero w/ a time characteristic of the preamp decay time
  - Because preamp's have long decays, the undershoot persists for a relatively long time
  - Another pulse can be superimposed on the undershoot and an error will be induced in its amplitude (= pulse pile-up?)

- Regarding the output pulse from the CR network for a step input as an input pulse w/ a finite decay for the CR network in the shaping amp, the overall transfer function is then
  \[
  \frac{s\tau_{PA}st_{SA}}{1+s\tau_{PA}}\frac{s\tau_{SP}}{1+s\tau_{SP}}
  \]
  - There are two poles
  - The step input becomes an output pulse w/ undershoot
Pole-zero cancellation is a technique to restore the simple exponential output w/o undershoot by adding a parallel resistance $R_{pz}$ w/ the capacitor of the CR network in the shaping amp. Then, the modified transfer function becomes

$$\frac{\tau_{PA}(1+sR_{pz}C_{SA})s\tau_{SA}}{(1+s\tau_{SA})(R_{pz}C_{SA}\tau_{PA}+R_{pz}C_{SA}+\tau_{PA})}$$

- If $R_{pz} = \frac{\tau_{SA}}{C_{SA}}$, the transfer function reduces to $\frac{s}{s+k}$ where $k = \frac{\tau_{PA}+\tau_{SA}}{\tau_{PA}\tau_{SA}}$
  
  - Now, we have a single pole, ensuring that the shaping amp once again produces a simple exponential decay for a step input
  - $R_{pz}$ is chosen empirically in practice (i.e. potentiometer)
Baseline shift

- Since, in ac-coupled circuits, the average dc voltage of the right of the capacitor of the CR network must be zero, the baseline must be suppressed below the true zero level, and the amount by which this baseline is depressed below true zero is called the "baseline shift"
  - Reducing the pulse amplitude

- No adequate compensation can be carried out for the pulse train, in which both the amplitude and spacing are variable, from detectors, because the degree of baseline shift is not constant
  - Being severer at high counting rates

- Remedy: use the bipolar pulse shaping
  - Bipolar pulses w/ equal area btwn (+)ive & (-)ive lobes can be passed by a capacitor w/o alteration of the baseline
  - Poorer SNR compared w/ monopolar pulses
  - dc coupling is alternative but impractical
• ac-coupled circuits
  – Coupling capacitor
  – Can be allowed to interchange $R_L$ w/o affecting the preamp input

• dc-coupled circuits
  – Generally better noise performance due to no coupling capacitor
  – Detector must be isolated from ground
  – Changing $R_L$ may affect the input stage of characteristics
• Baseline restorer
  – Active electronic circuit
  – Should be placed near the end of signal chain to avoid further ac-coupling
    • Usually at the output stage of linear amplifiers
  – Closing the switch, after the duration of each pulse, restores the output voltage to zero
    • Time constant $= (R + R_0)C$
Bipolar shaping networks

- **CR-RC-CR network**
  - Some baseline shift can be remained when the areas of the two lobes are not equal
  - Most useful at high counting rates
  - Poor SNR

- **Single delay line (SDL) network**

- **Double delay line (DDL) network**
Summary of pulse shaping

• Preamp
  – Integrating the current pulse from detectors w/ a long \( \tau \) circuit, & then producing a long tail pulse
  – The amplitude of the long tail pulse is sometimes too small to be counted directly and causes stability problem due to pulse pile-up (not adequate at high counting rates)

• Linear shaping amp
  – Shaping the long tail pulse to a much shorter width and increasing its amplitude by a factor given by its gain
  – Usually 0–10 V positive
  – Typically 0.5–10 \( \mu \)s widths
Single-channel analyzer

- Integral discriminator
  - **Linear-to-logic converter**: Producing a logic output pulse only if the linear input pulse amplitude exceeds (or *the leading edge of input pulse crosses*) a discrimination level
  - Discrimination level is adjustable (e.g. set just above the system noise level)
• Differential discriminator (= single-channel analyzer)
  – Employing two indep. discriminators (lower-level & upper-level discriminators)
  – Producing a logic pulse only if the input linear pulse amplitude lies btwn the two levels
  – Able to selectively measure one type or energy of radiation in the presence of other radiations

• Counters
  – Sometimes called scalers (as a historic anomaly)
  – Recording # of logic pulses over a fixed period of time by using *digital registers*
  – Operated in either mode: preset time & preset count
Multichannel analyzer

• Differential pulse height distribution or pulse height spectrum
  – A continuous curve that plots the value of $dN/dH$ vs. the value of the pulse height $H$
    • $dN/dH$ = the differential # of pulses observed w/i a differential increment of pulse height $H$
  – Discrete approximation: $\Delta N/\Delta H$ vs. $H$
    • $\Delta N/\Delta H$ = the discrete # of pulses observed in a small but finite increment of pulse height $H$
    • $\Delta H$ = the increment in $H$ = window width or channel width

• Mainly consisting of:
  – Analog-to-digital converter (ADC): from pulse amplitude to digital number
  – Memory
Analog pulse processing

Digital pulse processing
• Input gate
  – To block pulses from reaching the ADC during digitizing a previous pulse ("busy")
  – **Dead time** some fraction of the input pulses will be lost

• Live time clock
  – Internal clock whose output pulses are routed thru the same input gate and are stored in a special memory location for **correcting** dead time
  – # of clocks recorded in the memory is a measure of *live time* of MCA (fraction of clocks blocked by the input gate = fraction of input pulses blocked)

• SCA
  – Open the **linear gate** for a selected band of input pulse amplitudes; hence reducing # of conversions in ADC (by rejecting uninteresting input pulses that are either smaller than LLD [e.g. small noise pulses] or larger than ULD [e.g. large pulses beyond the range of interest])

• External gate
  – To prevent recording of pulses that are distorted in the prior stage of analog pulse processing