Projection Radiography

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Projection radiography

- Most commonly used method of medical imaging utilizing x rays, called conventional radiography
- Regarding shadow cast by a semitransparent body illuminated by x rays
- Representing a projection of the 3-D volume of the body onto a 2-D imaging surface
  ⇒ limiting depth resolution; hence limiting contrast
  (conspicuity: "hide" important lesions)
- Representing the “transmission” of the x-ray beam thru the patient, weighted by the integrated loss of beam E due to scattering & absorption in the body

- To screen for pneumonia, heart disease, lung disease, bone fractures, cancer & vascular disease

- Overall imaging conditions of projection radiographic system
  - Exposure time ~0.1 sec
  - Image size 14 × 17 in.
  - Exposure level 30 mR of a chest radiograph (= 1/10 of the annual bgn dose)
Instrumentation

- X-ray tube
- Filters/collimators
- Grids
- Detectors
X-ray tubes

- **Cathode assembly**
  - Filament
    - Thin thoriated* tungsten wire
      (*thoriate: add thorium or thorium dioxide to increase thermionic flow)
    - Typ. filament current = 3–5 A at 6–12 V
    - Thermonic emission of $e^-$'s, accelerated toward the *anode* producing tube current, referred to as the **mA** (typ. 50–1,200 mA)
    - Filament current $\rightarrow$ heat (resistance) $\rightarrow$ # of discharged $e^-$'s $\rightarrow$ tube current
  - Focusing cup
    - Small depression in the cathode containing the filament
    - To help focus the $e^-$ beam toward a particular spot on the anode
      $\Rightarrow$ *Focal point*: a bevelled edge of the anode disk
• Anode

– Made from molybdenum on which a rhenium-alloyed tungsten target is coated
– Mo in the target area for mammography x-ray tube
– Tube voltage kVp (typ. 30–150 kVp)
– Generating “characteristic” & “bremsstrahlung” x rays
– ~1% of conversion efficiency (~99% → heat)

– Stator electromagnets induce the rotors to rotate the anode
  • To avoid melting the anode target area
  • 3,200–3,600 rpm

• Exposure \( \iff mA \times s \) (exposure time) = mAs during the applied kVp

– Controlled by fixed timer circuit or automatic exposure control (AEC) timer
  • Controlling the overall exposure determined by the duration of the applied kVp

– Fixed timer circuit
  • Silicon-controlled rectifier (SCR) switch timed by a microprocessor
  • Timing accuracy ~ 0.001 s
  • Radiologists control the “mA” & the exposure “time” directly, hence determining the “mAs”

– AEC timer
  • 5-mm-thick parallel plate ionization chamber placed between the patient table & the imager
  • Voltage signal from ion chamber triggers the SCR, which shuts off the tube voltage
  • Radiologists control the “mAs” & the exposure time is determined automatically by the AEC
  • Set max. time to prevent accidental overdose due to that AEC circuit malfunctions or the ion chamber is missing or incorrectly positioned
Filtration & restriction

- **Filtration**
  - **Process of absorbing low-E x-ray photons before they enter the patient**
  - X-ray spectrum: polyenergetic, bremsstrahlung + characteristic radiations
  - Very “undesirable” for low-E photons to enter the body
    - Almost entirely absorbed w/i the body → contributing to “patient dose” but not the image
  - **Inherent filtering**
    - Tungsten anode itself
    - Glass housing of the x-ray tube & the dielectric oil
    - Accentuated over time due to the aging of x-ray tubes (vaporization of W filaments)
  - **Added** filtering
    - Placing metal in the x-ray beam outside of the tube
      - Al (1–2 mm thick) typically
      - Higher-E systems: Cu + Al (to attenuate the 8 keV characteristic x rays from Cu)
    - Additional 1.0 mm Al/Eq of filtering by the “silvered mirror” placed w/i the “collimator”
  - **Beam hardening**
    - The increase in the beam’s “effective energy” as a progressive shift in the position of the spectrum “to the right” due to filtering

Note that the National Council on Radiation Protection & Measurements (NCRP) recommends a min. filtration of 2.5 mm Al/Eq at the exit port of the x-ray tube for the reduction of patient dose.
Example

For radiography systems operating above 70 kVp, the NCRP recommends a min. total filtration of 2.5 mm Al/Eq at the exit port of the x-ray tube. Although such filtration reduces high-energy as well as low-energy x-rays, thus requiring longer exposure times to properly expose the x-ray film, the overall dose to the patient is reduced because of the reduction in low-energy x-rays are absorbed almost entirely by the patient. At 80 kVp, what thickness of Cu would provide 2.5 mm Al/Eq of filtration? (Note: $\mu/\rho$ (Al) = 0.2015 cm²/g & $\mu/\rho$ (Cu) = 0.7519 cm²/g at 80 kVp; $\rho$ (Al) = 2.699 g/cm³, $\rho$ (Cu) = 8.960 g/cm³)

• Restriction beam
  – Process of absorbing the x rays outside a certain field of view
  – To avoid exposing parts of the patient that need not be imaged
  – To help reduce the deleterious effects of “Compton scatter”
  – Basic kinds of beam restrictors
    1. Diaphragms
       – Flat pieces of lead w/ holes
       – Simple & inexpensive
       – Fixed geometry used in dedicated systems that have only on purpose (e.g., chest imaging)
    2. Cones or cylinders
       – Fixed geometry & better performance
    3. Collimators
       – Variable diaphragms comprised of movable pieces of lead
       – Expensive but flexible
       – Usually two collimators: one near the tube & one farther away from the tube
         » “Mirror” btwn them to illuminate the FOV w/ an alignment grid
Compensation filters & contrast agents

- Attenuation: the process by which x rays are absorbed or redirected (scattered) w/i the body or other objects in the FOV
  - Different amounts of attenuation in body tissues \( \sim \mu, E \)
    \( \rightarrow \) differential attenuation \( \rightarrow \) contrast
- Sometimes need to artificially change the natural attenuation of the body prior to detecting x rays

- Compensation filters
  - Used when difficult to obtain images of subjects having big differences in attenuation over area because of the limited dynamic range of x-ray detectors
  - Specially shaped Al or leaded-plastic object for an x-ray detector w/ smaller dynamic range
  - Placed btwn the x-ray source & the patient or in some cases btwn the patient & the detector
- Contrast agents
  - Used when difficult to visualize different soft tissues due to insufficient intrinsic contrast
  - Chemical compounds to increase x-ray absorption w/i an anatomical region

- Utilizing the *K-edge absorption* effect
  - Leading to very high *differential* absorption btwn the contrast agent & the surrounding tissues; enhancing contrast

- Iodine (Z = 53): $E_K = 33.2$ keV
  - Intravascular injection or ingestion
  - Blood vessels, heart chambers, tumors, infections, kidneys, bladder

- Barium (Z = 56): $E_K = 37.4$ keV
  - Administered as a chalky "milkshake"
  - Gastrointestinal tract

- Air itself
  - "Opposite" type of contrast to that of I & Ba
  - e.g., "inflated" lungs
Grids, airgaps, & scanning slits

- Scattering process
  - Causing a random “fog” throughout the image; thereby reducing the contrast of image

- Reduction methods: grids, airgaps, & scanning slits

- Grids
  - Thin strips of Pb alternating w/ highly transmissive interspace mat’l (e.g., Al or plastic)
  - Linear, focused grid

  ![Diagram of a grid](image)

  - Grid ratio
    - A measure of the effectiveness of the grid for reducing scatter
    - \( \text{grid ratio} = \frac{h}{b} \)
      - \( h \) = height of the lead strips
      - \( b \) = spacing btwn the lead strips
      - 6:1–16:1 in conventional radiographic systems
      - Down to 2:1 in mammography systems
      - Higher grid ratio for thick body parts (e.g., abdomen or chest)
      - Lower grid ratio for thin body parts (e.g., extremities)

  - Grid spacing
    - Generally reported using its reciprocal, known as grid frequency
      - 60 cm\(^{-1}\) for conventional radiographic systems
      - 80 cm\(^{-1}\) for mammography systems
Note that grids with higher grid ratios (tall lead strips or fine lead strip spacing) are more capable of stopping off-axis radiation => Requiring higher patient dose to maintain a high-quality image.

Consider the tradeoff between primary & scatter radiations:

- **Grid conversion factor (GCF)**
  - Characterizing the amount of additional exposure required for a particular grid
  - \[ GCF = \frac{\text{mAs with the grid}}{\text{mAs without the grid}} \]
  - Ranging from 3 to 8
  - Use a grid when the tube voltage > 60 kVp (isotropic scattering for low-E photons); when imaging a body part thicker than 10 cm

- **Stationary grids**
  - Introducing visible artifacts

- **Potter-Bucky diaphragm**
  - Moving the grid 2 to 3 cm during exposure in a linear or circular path

- **Airgaps**
  - Leaving an airgap between the patient & the detector \(\rightarrow\) an effective means of scatter rejection
  - Cons:
    - Increased geometric magnification
    - Blurring or unsharpness due to x-ray focal-spot size effect

- **Scanning slits**
  - Placed in front of and/or in back of the patient
  - Providing greater than 95% scatter reduction
  - More complex & costly system
  - Longer exposure times
Film-screen detectors

Direct exposure of x-rays to a photographic film
→ About 1 to 2% of the x rays are stopped by the film (⇒ detection efficiency)
→ Requiring an unnecessarily large x-ray dose to the patient
→ A very inefficient way to create a photograph
→ Use of intensifying screens on both sides of the film, but an additional image blurring due to light scattering should be considered

• Intensifying screens
  – Base
    • Provided for mechanical stability
    • Somewhat flexible to be pushed tightly against the film
    • Typ. made of polyester plastic
  – Reflective layer
    • Reflecting light from the phosphor back into the film rather than getting lost in the base
    • Typ. about 25 μm thick
    • Made of magnesium oxide (MgO) or titanium dioxide (TiO₂)
  – Protective coating
    • Applied to film side of the screen to protect it from repeated film loading & unloading
Phosphors
- Converting x rays into light
- Luminescent materials
  - Fluorescence ≤ 1 × 10⁻⁸ s of the excitation
  - Phosphorescence in which light emission can be delayed
    ⇒ Causing "afterglow" ⇒ motion artifact, image lag
- Should be highly x-ray attenuating
  - high Z (so large μ)
- Should emit many light photons for every x-ray photon that is stopped
  - High conversion efficiency
    » A measure of the number of light photons emitted per incident x-ray photon
    » Typ. btwn 5 to 20% depending on the type of phosphor & its thickness
    » ~10³ light photons per incident 50-keV x-ray photon
    » Speed of screen
      » "Faster" if the conversion efficiency is higher
- Calcium tungstate (CaWO₄) discovered by Thomas Edison
- Rare earth phosphors in the late 1970s (terbium-doped gadolinium oxysulfide, Gd₂O₂S:Tb)

Radiographic film
- Optical film to capture the optical image created w/i the screens that sandwich the film
- Common size in US: 14 × 17, 14 × 14, 10 × 12, 8 × 10, & 7 × 17 in.
- Image quality depends on optical properties & details of chemical development

Radiographic cassette
- A holder for two intensifying screens & the film "sandwiched" btwn
- One side "radiolucent", the other "a sheet of lead foil"
X-ray image intensifiers

- XRII in fluoroscopy requires low-dose, real-time projection radiography

![Diagram of X-ray image intensifier](image)

- **Input window**
  - Aluminum or titanium w/ 0.25–0.5 mm thickness
  - \( \Rightarrow \) minimal loss of x-ray photons but capable of holding a vacuum

- **Input phosphor**
  - Typ. CsI(Na) w/ 0.5 mm thickness
  - 15–40 cm in diameter
  - 0.5-mm-thick aluminum reflector

- **Photocathode**
  - Generating free \( e^- \)s w/i the vacuum tube

- **Dynodes**
  - A series of electrodes to accelerate \( e^- \)s
  - Shaping \( e^- \)s into an (inverted) \( e^- \) intensity image
  - Alternating voltage profile to provide variable image magnification

- **Anode**
  - 25–35 kV relative to the cathode
Image Formation

• Basic imaging equation
  Consider a particular line segment thru the object starting at the x-ray origin & ending on the detector plane at point \((x, y)\):

$$I(x, y) = \int_{0}^{E_{\text{max}}} S_{0}(E')E'e^{-\int_{0}^{(x,y)} \mu(\tau, E, x, y) ds} dE'$$

- \(S_{0}(E)\) = the spectrum of the incident x-ray
- \(s\) = distance from x-ray origin to the point \((x, y)\) on the detector plane = \(r\)
Geometric effects

→ Undesirable (“multiplicative”) effects due to the natural x-ray beam divergence

- Inverse square law
  - Dependency of the net flux of photons (i.e., photons per unit area) on \(1/r^2\)
  - Intensity at the origin of the detector assuming no attenuation
    \[ I_0 = \frac{I_s}{4\pi d^2} \]  
    \(I_s\) = beam intensity [# of photons] integrated over a small sphere surrounding the source
    \(d\) = the source-to-detector distance
  - Intensity at \(r = r(x, y)\) on the detector
    \[ I_r = \frac{I_s}{4\pi r^2} \]  
    \(I_r < I_0\) because \(r > d\)
  - \(\cos^2\theta\) drop-off of x-ray intensity due to the inverse square law
    \[ I_r = \frac{I_s}{4\pi r^2} = \frac{4\pi d^2 I_0}{4\pi r^2} = I_0 \frac{d^2}{r^2} = I_0 \cos^2\theta \]  
    ⇒ Resulting in “false” object attenuation in a circular pattern around the detector origin

Example

The inverse square law has a very practical use in radiography. Suppose an acceptable chest radiograph was taken using 30 mAs at 80 kVp from 1 m. Suppose that it was now requested that one be taken at 1.5 m at 80 kVp. What mAs setting should be used to yield the same exposure?
• Obliquity

- Decreasing the beam intensity away from the detector origin when the detector surface is not orthogonal to the direction of x-ray propagation
- Implying that x rays pass through a larger area on the detector
  - Lowering x-ray flux
  - Resulting in a lower measured x-ray intensity on the detector surface

\[ I_d = I_0 \cos \theta \]

\( \Leftarrow \) oblique area \( A_d = \frac{A}{\cos \theta} \), where \( A \) = the area orthogonal to the beam direction

• Beam divergence & flat detector

- Reduction in beam intensity due to
  1) The inverse square law effect
  2) Obliquity

\[ I_d(x, y) = I_0 \cos^3 \theta \]

• Anode heel effect

- Nonuniform x-ray intensity distribution along the cathode-anode direction, but no variation orthogonal to the cathode-anode direction

\( \Rightarrow \) Stronger intensity in the cathode direction
  - \(~45%\) intensity variation in the cathode-to-anode direction

\( \Rightarrow \) Far outweighing the effects of obliquity & inverse square law (i.e., more important)

\( \Leftarrow \) Compensated by an x-ray filter that is thicker in the cathode direction
Example

Suppose a chest x-ray is taken at 2 yards using 14 inch by 17 inch film. What will be the smallest ratio $I_d/I_0$ across the film (assuming no object attenuation)?

- **Path length**
  - X-ray intensity at $(x, y) = (0, 0)$:
    $$I_d(0,0) = I_0 e^{-\mu L}$$
  - X-ray intensity at $(x, y) \neq (0, 0)$:
    $$I_d(x, y) = I_0 e^{-\mu L / \cos \theta}$$
  - Considering the $1/r^2$ & obliquity:
    $$I_d(x, y) = I_0 \cos \theta e^{-\mu L / \cos \theta}$$

  ⇒ Causing a shading artifact w/i a homogeneous (attenuation & thickness) object
  ⇒ Misleading as a different attenuation w/i the object or different object thickness
• **Depth-dependent magnification**
  
  – Consider the “stick” object of height $w$:
  
  $\Rightarrow w_z$ on the detector is different depending on the position $z$ of the object in FOV

  $$w_z = w \frac{d}{z} \implies \text{magnification } M(z) = \frac{d}{z}$$

  \rightarrow Different sizes for two object w/ the same size (judgment about relative sizes of anatomical features must be made with caution and knowledge)

  \rightarrow Difficulty in the “longitudinal study” at the same radiographic conditions & patient positioning

  \rightarrow **Depth-dependent** blurring

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**Example**

Consider imaging the rectangular prism defined by $\mu(x, y, z) = \mu_\text{a} \text{rect}(\frac{y}{W}) \text{rect}(\frac{x}{W}) \text{rect}(\frac{z-z_0}{L})$ where $\text{rect}(x) = \begin{cases} 1 & |x| \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$.

How is this rectangular prism portrayed in a radiograph?
• Imaging equation w/ geometric effects
  – Consider an idealized object $t_z(x, y)$:
    • Infinitesimally thin;
    • Located in a single plane $z$;
    • Capable of differentially attenuating x rays as a function of $x$ & $y$;
    ⇒ Can be regarded as a transmittivity

$$I_d(x, y) = I_0 \cos^3 \theta e^{-\mu / \cos \theta} \Rightarrow I_0 \cos^3 \theta t_d(x, y) \text{ at } z = d$$

$$\cos \theta = \frac{d}{\sqrt{d^2 + x^2 + y^2}}$$

– Considering the magnification at arbitrary $z$, where $0 < z \leq d$;

$$I_d(x, y) = I_0 \cos^3 \theta t_d(x/M(z), y/M(z))$$
or

$$I_d(x, y) = I_0 \left(\frac{d}{\sqrt{d^2 + x^2 + y^2}}\right)^3 t_d(\frac{xz}{d}, \frac{yz}{d})$$

⇒ Reasonable approximation only for relatively thin objects that have nearly the same $M(z)$ & no variation in attenuation in the $z$ direction

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Blurring effects

Due to “extended” sources & the intensifying screen

• Extended sources (= finite focal spot size)
  ⇒ Fuzziness both at the edge of the FOV & object boundaries

  – Size of a point hole located at range $z$ for the source w/ diameter $D$;

$$D' = \frac{d - z}{z} D = -\frac{m(z)}{z} D$$

$$m(z) = -\frac{d - z}{z} D = [-M(z) + 1]D$$

» source magnification

» Negative because it inverts the image of the source = $1 - M(z)$
– Source image for the source intensity distribution $s(x, y)$
  
  \[ I_d(x, y) = ks(x/m, y/m) \]

  - $m = \text{the source magnification for a point at range } z$
  - $k = \text{the amplitude scaling term}$

  » $\int \int ks(x/m(z), y/m(z))dx dy = \text{constant}$ (because the integrated intensity on the detector plane must remain constant regardless of the position of the point hole on the z-axis)

  » Fourier transform $\Rightarrow km^2(z)S(0,0) = \text{constant}$

  » $k \propto \frac{1}{m^2(z)}$

  $\Rightarrow$ The amplitude of blurring due to the extended source depends on the location of the object relative to the detector plane

  \[ \frac{s(x/m,y/m)}{m^2} \rightarrow S(0,0)\delta(x, y) \text{ as } m(z) \rightarrow 0 \]

  $\Rightarrow$ No loss of resolution or amplitude

– Spatial distribution of attenuating objects w/i a given z plane:

  \[ I_d(x, y) = \frac{\cos^2 \theta}{4\pi d^2} s(x/m, y/m) * t_z(x/M, y/M) \]
• Film-screen blurring

Consider an “additional convolution” w/ the film-screen impulse response function $h(x, y)$ assuming that light photons isotropically scatter:

$$I_d(x, y) = \frac{\cos^2 \theta}{4\pi d^2 m^2} s(x/m, y/m) * t_2(x/M, y/M) * h(x, y)$$

– Tradeoff btwn image resolution (or blur) & detector efficiency $\eta$
  • $\eta = \text{the fraction of photons captured by the detector on average}$
    $\Rightarrow \sim 0.3$ for CaWO$_4$

• Usually negligible MTF of the film compared w/ the intensifying screen
• Ignoring the image created directly on the film by x rays due to very poor $\eta$
  $\Rightarrow$ Only considering the image created by the light produced in the phosphors adjacent the film

X rays captured by the screen $\rightarrow$ converted to light photons & captured by the film $\Rightarrow$ a latent image (i.e., not visible) $\rightarrow$ a blackening of the film

• Characterizing film as a transformation btwn exposure to light & the degree of blackening of film $\Rightarrow$ optical density of the film
• Optical transmittivity = the fraction of light transmitted thru the exposed film
  \[ T = \frac{I_t}{I_i} \]
  - \( I_t \) = the irradiation of the transmitted light [energy/area/sec]
  - \( I_i \) = the irradiation of the incident light [energy/area/sec]

• Optical opacity = \( T^{-1} \)

• Optical density = the common logarithm of the optical opacity
  \[ D = \log_{10} \frac{I_i}{I_t} \]
  → Characterizing “how black” the film in on a logarithmic scale
  → Usable when \( 0.25 \leq D \leq 2.25 \)
  → Best discrimination of shading gray when \( 1 < D < 1.5 \)

• H&D (Hurter & Driffield) curve
  - S-shaped curve:
    - Low D toe @ low exposure / “linear” portion / high D shoulder @ high exposure
  - Base fog = non-zero optical density even in the absence of exposure
  - In the linear region:
    - \( D = \Gamma \log_{10} \frac{X}{X_0} \)
      - \( X_0 \) = the exposure when the linear region would hit the horizontal axis (\( D = 0 \))
      - \( \Gamma \) = the slope of the H&D curve in the linear region (\( \Rightarrow \) “contrast”)
        » Called the “film gamma”, typ. 0.5–3
        » \( \Gamma \uparrow \Rightarrow \) contrast ↑ but latitude ↓
    - Latitude = the range of exposures over which the H&D curve is linear (\( \Rightarrow \) “dynamic range”)
    - Speed of film = the inverse of the exposure at which “\( D = 1 + \) fog level”
Noise & Scattering

• Signal-to-noise ratio (SNR)

Consider the detector intensities from a rectangular object assuming unity magnification & infinitesimal source size:

- Local contrast

\[ C = \frac{I_t - I_b}{I_b} \]

- Addition of noise due to the random fluctuation in # of photons arriving in each small area of detector
  ⇒ Called “quantum mottle”
  → Responsible for the impression of detector measurements of x-ray intensity
SNR = \frac{I_I - I_b}{\sigma_b} = C \frac{I_b}{\sigma_b}

- X-ray intensity in the number of photons using the effective energy (hν) for polyenergetic beam:
  - \( I = \frac{N hν}{AΔt} \)
- Average background intensity:
  - \( I_b = \frac{N_b hν}{AΔt} \)
- Variance of the number of photons per burst per area \( A \) in the background:
  - \( \sigma_b^2 = N_b \left( \frac{hν}{AΔt} \right)^2 \)

Then, the local SNR:
- \( \text{SNR} = C \sqrt{N_b} = C \sqrt{ΦARtη} \)
  - \( Φ \) = the number of photons per Roentgen per cm²
  - \( A \) = the unit area
  - \( R \) = the body’s radiation exposure in Roentgens
  - \( t \) = the fraction of photons transmitted thru the body
  - \( η \) = the detector efficiency

- How to improve the visibility of a particular structure in a radiograph?
  - Increase the contrast of the structure
    - Change x-ray energy (kVp)
    - Use a contrast agent
    - Use dual-energy techniques
  - Increase the number of photons used in the visualization or analysis
    - Increase mAs
    - Increase x-ray energy (more penetration)
    - Use a large-pixel detector
    - Use a more efficient detector
Quantum efficiency & detective quantum efficiency

- Quantum efficiency
  - Probability that a single photon incident upon the detector will be detected

- Detective quantum efficiency
  - Considering the transformation of SNR from a detector’s input to its output
  - A measure of the degradation in the SNR due to the detection process
  - The fraction of photons that are detected “correctly”
  - DQE ≤ QE ≤ 1
    - DQE = \( \left( \frac{SNR_{out}}{SNR_{in}} \right)^2 \)
      - \( SNR_{in} \) = the intrinsic SNR of the incident radiation
      - \( SNR_{out} \) = the SNR of the measured quantity

Example

Consider a hypothetical detector having QE = 0.5 and the ability to perfectly localize every photon that is stopped by the detector. What is the DQE of this detector?
Example

Suppose that an x-ray tube is set up to fire \( n \) 10,000-photon bursts at a detector and the detector’s output \( x \) is recorded as \( x_i, i = 1, \cdots, n \). Suppose that the mean and variation of \( \{x_i\} \) is found to be \( \bar{x} = 8,000 \) and \( \sigma_x^2 = 40,000 \), respectively. What is the DQE of this detector?

• Compton scattering
  → Resulting in a decrease in image contrast & a decrease in SNR

  – Effect on image contrast

  Recall the local contrast \( C = \frac{l_e - l_b}{l_b} \)
  
  • Assuming that CS adds a constant intensity \( I_s \) to both target & background intensities
  
  \[ C' = \frac{(l_e + I_s) - (l_b + I_s)}{l_b + I_s} = C \frac{l_b}{l_b + I_s} = C \frac{1}{1 + I_s/I_b} \]
  
  ⇒ Scatter reduces contrast by the factor \( 1/(1 + I_s/I_b) \)
  
  – \( I_s/I_b \) called “scatter-to-primary ratio”

• SNR w/ scatter
  
  – \( \text{SNR}' = \frac{l_e - l_b}{\sigma_b} = C \frac{l_b}{\sigma_b} = C \frac{N_b}{\sqrt{N_b + N_z}} = C \frac{\sqrt{N_b}}{\sqrt{1 + N_z/N_b}} \)
  
  – \( \text{SNR}' = \frac{\text{SNR}}{\sqrt{1 + l_b/l_b}} \)
Example

Suppose 20% of the incident x-ray photons have been scattered in a certain material before they arrive at detectors. What is the scatter-to-primary ratio? By what factor is the SNR degraded?