Quantities for Describing Radiation Interactions

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References


Nonstochastic quantities for describing the interactions of the radiation field with matter (in terms of expectation values for the infinitesimal sphere at the point of interest)

1. Kerma $K$
   - The first step in energy dissipation by indirectly ionizing radiation (i.e., energy transfer to charged particles)

2. Absorbed dose $D$
   - The energy imparted to matter by all kinds of ionization radiations, but delivered by the charged particles

3. Exposure $X$
   - $X$- & $\gamma$-ray fields in terms of their ability to ionize air
Kerma

- Relevant only for
  - Fields of indirectly ionizing radiations (e.g., photons or neutrons)
  - Any ionizing radiation source distributed within the absorbing medium

- Energy *transferred*

\[ \varepsilon_{tr} = (R_{in})_u - (R_{out})^{nonr}_u + \sum Q \]

- \( \varepsilon_{tr} \) = energy transferred (*stochastic* quantity)
- \( (R_{in})_u \) = radiant energy\(^\dagger\) of uncharged particles entering \( V \)
- \( (R_{out})^{nonr}_u \) = radiant energy of uncharged particles leaving \( V \), *except* that which originated from radiative losses\(^\ddagger\) of KE by charged particles while in \( V \)
- \( \sum Q \) = net energy derived from rest mass in \( V \)
  - positive when \( \Delta m \downarrow (m \rightarrow E) \)
  - negative when \( \Delta m \uparrow (E \rightarrow m) \)

\(^\dagger\) The energy of particles (excluding rest energy) emitted, transferred, or received

\(^\ddagger\) Conversion of charged-particle KE to photon energy through either bremsstrahlung or *in-flight* annihilation of positrons
\[ K \equiv \frac{d\epsilon_{tr}}{dm} \]

- The *expectation* value of the energy transferred to charged particles per unit mass at a point of interest, including radiative-loss energy but excluding energy passed from one charged particle to another
- Simply, the KE received by charged particles in the specified finite volume \( V \)
- The kerma for x- or \( \gamma \)-rays consists of the energy transferred to electrons & positrons per unit mass of medium
- \( 1 \) Gy = \( 1 \) J/kg = \( 10^2 \) rad = \( 10^4 \) erg/g

- **For monoenergetic photons**

\[ K = \Psi \left( \frac{\mu_{tr}}{\rho} \right)_{E,Z} \]

- \( \frac{\mu_{tr}}{\rho} \) = mass energy-transfer coefficient (depending on \( E \) & \( Z \))
- \( \mu_{tr} \) = linear energy-transfer coefficient
For spectral photons

\[ K = \int_0^{E_{\text{max}}} \Psi'(E) \left( \frac{\mu_{\text{tr}}}{\rho} \right)_{E,Z} dE \]

An average value of mass energy-transfer coefficient for the spectrum \( \Psi'(E) \) is given by

\[ \left( \frac{\mu_{\text{tr}}}{\rho} \right)_{\Psi'(E),Z} = \frac{K}{\Psi} = \frac{\int_0^{E_{\text{max}}} \Psi'(E) \left( \frac{\mu_{\text{tr}}}{\rho} \right)_{E,Z} dE}{\int_0^{E_{\text{max}}} \Psi'(E) dE} \]

Kerma rate

\[ \dot{K} = \frac{dK}{dt} = \frac{d}{dt} \left( \frac{d\epsilon_{\text{tr}}}{dm} \right) \]
Components of kerma

\[ K = K_c + K_r \]

- \( K_c \) = kerma due to collision interactions (local or nearby the charged-particle track)
  - Coulomb-force interactions with atomic electrons (ionization & excitation)
- \( K_r \) = kerma due to radiative interactions (remote or far away from the charged-particle track)
  - radiative interactions with the Coulomb force field of atomic nuclei
  - bremsstrahlung, in-flight annihilation

Net energy transferred

\[ \epsilon_{tr}^n = (R_{in})_u - (R_{out})_{u nonr} - R_u^r + \sum Q = \epsilon_{tr} - R_u^r \]

- \( R_u^r \) = radiant energy emitted as radiative losses by the charged particles (which themselves originated in \( V \))
Collision kerma

\[ K_c = \frac{d\epsilon_{tr}^n}{dm} \]

- The \textit{expectation} value of the \textit{net} energy transferred to charged particles per unit mass at a point of interest, \textit{excluding} both radiative-loss energy and energy passed from one charged particle to another.

Radiative kerma

\[ K_r = \frac{dR_{tr}}{dm} \]
For monoenergetic photons

\[ K_c = \Psi \left( \frac{\mu_{en}}{\rho} \right)_{E,Z} \]

- \( \frac{\mu_{en}}{\rho} \) = mass energy-absorption coefficient (depending on \( E \) & \( Z \))
  - \( \frac{\mu_{en}}{\rho} \approx \frac{\mu_{tr}}{\rho} \) for low \( Z \) and \( E \) (where radiative losses are small)
- \( \mu_{en} \) = linear energy-absorption coefficient

<table>
<thead>
<tr>
<th>( E_{\gamma} ) (MeV)</th>
<th>( \frac{\mu_{tr} - \mu_{en}}{\mu_{tr}} \times 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Z = 6 )</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>3.5</td>
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</table>
Absorbed dose

- Relevant for
  - All types of ionizing radiation fields
  - Any ionizing radiation source distributed within the absorbing medium

- Energy *imparted*

\[ \epsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q \]

- \( \epsilon \) = energy imparted (*stochastic* quantity)
- \( (R_{out})_u \) = radiant energy of all the uncharged radiation leaving \( V \)
- \( (R_{in})_c \) = radiant energy of the charged particles entering \( V \)
- \( (R_{out})_c \) = radiant energy of the charged particles leaving \( V \)
- \( \sum Q \) = net energy derived from rest mass in \( V \)
\[ D = \frac{d\epsilon}{dm} \]

- The *expectation* value of the energy imparted to matter per unit mass at a point
- The same dimension & units with \( K \)
- Simply, the energy per unit mass to produce any effects attributable to the radiation (the most important quantity in radiological physics)
- Impossible to write a relationship \( D \) and \( \Psi \) of indirect radiation
- 1 Gy = 1 J/kg = 10² rad = 10⁴ erg/g

- **Absorbed dose rate**

\[ \dot{D} = \frac{dD}{dt} = \frac{d}{dt} \left( \frac{d\epsilon}{dm} \right) \]
Comparative example 1 of $\epsilon$, $\epsilon_{tr}$, & $\epsilon_{tr}^n$

**FIGURE 2.1a.** Illustration of the concepts of energy imparted, energy transferred, and net energy transferred for the case of a Compton interaction followed by bremsstrahlung emission (Attix, 1983).

Attix Fig. 2.1a
Comparative example 2 of $\epsilon$, $\epsilon_{tr}$, & $\epsilon_{tr}^n$

**FIGURE 2.1b.** Example involving $\gamma$-ray emission, pair production, and positron annihilation (Attix, 1983).
If the positron in Attix Fig. 2.1b had been annihilated in flight when its remaining KE was $T_3$, what are the values of $\epsilon$, $\epsilon_{tr}$, & $\epsilon_{tr}^n$?
Exposure

- Nonstochastic quantity defined only for x-ray & γ-ray photons

\[ X = \frac{dQ}{dm} \]

- The absolute value of the total charge \( dQ \) of the ions (of one sign) produced in air when all the electrons (negatrons & positrons) liberated by photons in air of mass \( dm \) are completely stopped in air
- The ICRU says that "the ionization arising from the absorption of bremsstrahlung emitted by the electrons is not to be included in \( dQ \)"
  - Also, the radiative losses through in-flight annihilation positrons
- Simply, the ionization equivalent of the \( K_c \) in air for x- & γ-rays
**W-value \( \bar{W} \)**

- See *Attix 12* for more details
- The mean energy expended in a gas per ion pair formed

\[
\bar{W} = \frac{\sum T_i (1 - g_i)}{\sum N_i (1 - g'_i)}
\]

- All the KE spent by electrons in collision interactions
- All the ion pairs produced in collision interactions by electrons

- \( T_i \) = initial KE of the \( i \)th electron (or positron)
- \( g_i \) = fraction of \( T_i \) that is spent by the particle in radiative interactions along its full path in air
- \( 1 - g_i \) = fraction spent in collision interactions
- \( N_i \) = total number of ion pairs that are produced in air by the \( i \)th electron of energy \( T_i \)
- \( g'_i \) = fraction of the ion pairs that are generated by the photons resulting from radiative loss interactions
- \( 1 - g'_i \) = fraction of the ion pairs produced by collision interactions that occur along the particle track

- **Not** count the energy going into radiative losses, **nor** the ionization produced by the resulting photons
• eV/ion pair
  – 33.97 eV/ip for x- & γ-rays in air

\[ \bar{W}_{air} = \frac{33.97 \text{ eV (or electron)} }{1.602 \times 10^{-19} \text{ C/electron}} \times 1.602 \times 10^{-19} \text{ J/eV} = 33.97 \frac{\text{J}}{\text{eV}} \]

• Constant values for each gas, independent of photon E, for x- & γ-ray energies above a few keV
• Convenient for relating \((K_c)_{air}\) and \(X\)

- Exposure rate

\[ \dot{X} = \frac{dX}{dt} \]
**X to \( \Psi \)**

- **For monoenergetic photons**

  \[
  X = \Psi \left( \frac{\mu_{en}}{\rho} \right)_{E, \text{air}} \left( \frac{e}{W} \right)_{\text{air}} = (K_c)_{\text{air}} \left( \frac{e}{W} \right)_{\text{air}} = \frac{(K_c)_{\text{air}}}{33.97}
  \]

- \( 1 \text{ R} = \frac{1 \text{ esu}}{0.001293 \text{ g}} \times \frac{1 \text{ C}}{2.998 \times 10^9 \text{ esu}} \times \frac{10^3 \text{ g}}{1 \text{ kg}} = 2.580 \times 10^{-4} \text{ C/kg} \)

- **Conversion factors**
  - \( X \) (in C/kg) = \( 2.58 \times 10^{-4} X \) (in R)
  - \( X \) (in R) = \( 3876 X \) (in C/kg)

- **For spectral photons**

  \[
  X = \int_0^{E_{\text{max}}} \Psi'(E) \left( \frac{\mu_{en}}{\rho} \right)_{E, \text{air}} \left( \frac{e}{W} \right)_{\text{air}} \text{ d}E \approx \left( \frac{e}{W} \right)_{\text{air}} \int_0^{E_{\text{max}}} \Psi'(E) \left( \frac{\mu_{en}}{\rho} \right)_{E, \text{air}} \text{ d}E
  \]
Significance of exposure

1. $\Psi$ is proportional to $X$ for any given photon energy or spectrum
2. The measurement of $X$ may estimate the effects of x- or $\gamma$-ray in tissue because air is an approximately "tissue-equivalent" material ($Z_{\text{air}} \approx Z_{\text{muscle}}$)
3. The value of $K_c$ in muscle, per unit $X$, is nearly independent of photon $E$
4. One can characterize a photon field at a point

**FIGURE 2.2a.** Ratio of mass energy-absorption coefficients for muscle and water relative to air. [Based on data of Hubble, as given by Evans (1968) for $h\nu > 0.15$ MeV, and by Greening, (1972) for $h\nu \leq 0.15$ MeV.]

**FIGURE 2.2b.** Ratio of mass energy-absorption coefficients for acrylic plastic and compact bone relative to air. Acrylic plastic ($C\text{H}_2\text{O}_3$) is variously called Lucite, Plexiglas, and Perspex. Data sources as in Fig. 2.2a.
Other quantities for radiation protection

- **Quality factor \( Q \)**
  - *Weighting factor* to provide an estimate of the relative human hazard of different types & energies of ionizing radiations
  - Dimensionless
  - Determined from the experimental *relative biological effectiveness* (RBE) & the *unrestricted linear energy transfer* \( (L_\infty) \) or the *collision stopping power*

![Graph showing the relationship between quality factor \( Q \) and collision stopping power in water, as recommended by the ICRP (1971).](Image)
- **Dose equivalent** $H$

  $$H \equiv DQ$$

  - Defined at a point (i.e., a *point* quantity)
  - Sievert, 1 Sv = 1 J/kg
  - 1 rem = $10^{-2}$ J/kg (equivalently to "rad")
  - Not strictly a physical quantity

- **Equivalent dose** $H_{T,R}$

  $$H_{T,R} = D_{T,R}w_R$$

  - Equivalent dose in an organ or in tissue $T$ due to radiation $R$
  - Not a point quantity but an *average* over a tissue or organ
  - $H_T = \sum_R H_{T,R} = \sum_R D_{T,R}w_R$
  - Not a measurable quantity

- **Effective dose**

  $$E = \sum_T H_T w_T$$

  - Not a measurable quantity